

# A Model of Anticipated Consumption Tax Changes\*

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## Abstract

This paper studies household spending responses to anticipated changes in the consumption tax. To do so, I construct a life-cycle heterogeneous-agent general equilibrium model with durables. The model features a wedge in durable transactions that reflects the actual consumption tax system: households pay the tax when buying the durables but do not receive the tax when selling them. There are three main findings. First, the baseline model reproduces an empirically consistent dynamic pattern of tax elasticity of the taxable spendings. Second, I find that life-cycle is a key component to match the level of tax elasticity of durable spending. Third, the baseline model generates smaller stockpiling of durables based on realistic motive than a model without the wedge. I then use the model for two counter-factual experiments. The first counter-factual experiment finds that the effect of a consumption tax cut is not symmetric to the tax hike. The second counter-factual experiment which compares a one-time tax hike and a multiple-times tax hike shows the multiple-times tax hike scheme generates smaller welfare cost than one-time tax hike.

**JEL code:** D15, E21, E62, H31.

**Keywords:** Durables, Anticipated Consumption Tax Change, Tax Wedge, Stockpiling, Life-cycle, Tax Elasticity.

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## 1. Introduction

Preannounced consumption tax hikes cause salient intertemporal substitution of household expenditures. Households largely increase the expenditures right before the consumption tax hikes and household expenditures plummet afterwards. From a policy perspective, the use of the preannounced future consumption tax hikes has been proposed as a household expenditures stimulus policy tool, labeled unconventional fiscal policy.<sup>1</sup> As such, understanding the effect and the mechanism of consumption tax changes on household expenditures is of the significant importance, particularly in a low interest rate environment when conventional monetary policy loses the efficacy.

The intertemporal substitution due to the consumption tax changes becomes even stronger in the presence of consumer durables. Many empirical studies show that the effects of the consumption tax changes are prominent in long-lasting goods or durable goods (e.g., [Cashin and Unayama 2016](#); [D’Acunto et al. 2016, 2018](#); [Baker et al. 2019, forthcoming](#); [Nguyen et al. 2020](#)). Therefore, it is a natural extension of a macroeconomic model to incorporate the durable goods into the analysis of the consumption tax changes.

Motivated by the empirical studies, this paper studies the effects of the anticipated future consumption tax changes on household expenditures using a life-cycle heterogeneous-agent general equilibrium model with consumer durables. A remarkable feature of my baseline model is a *tax wedge* which reflects the actual tax system. That is, the tax wedge is introduced to take into consideration that households pay the consumption tax when buying the durables but do not receive the tax when selling them. Since this is the case internationally, the tax wedge is a qualitatively *necessary* friction to analyze the consumption tax in a model of the durables.

Furthermore, for the quantitative analysis with the model to be credible, the model must be disciplined by key empirical regularities. The most important empirical moment for this analysis is the tax elasticity of household expenditures, since it measures how much the households respond to the consumption tax changes. I find that the tax wedge plays quantitatively central roles in reproducing the empirically consistent household expenditure responses for the following two reasons.

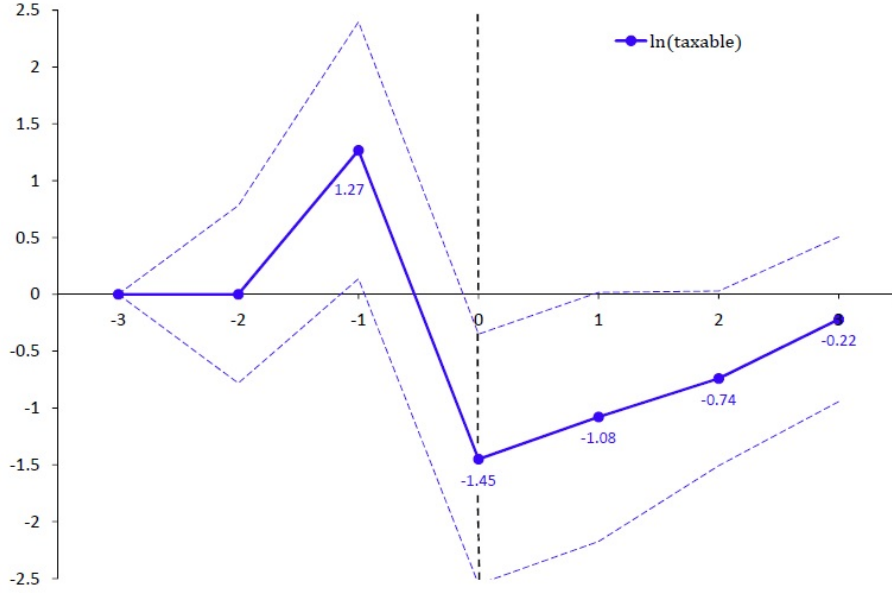
First, the tax wedge helps account for the estimated dynamic pattern of the taxable spendings, reported in [Baker et al. \(forthcoming\)](#). As [Figure 1](#) shows, [Baker et al. \(forthcoming\)](#) find that the elasticity of taxable spendings spike up right before the tax hike, plummet in the period of the tax hike and gradually recover to the new stationary states.<sup>2</sup> I find that the baseline model with

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<sup>1</sup>The unconventional fiscal policy is, for example, proposed by [Feldstein \(2002\)](#) and [Hall \(2011\)](#), and extensively studied by [Correia et al. \(2013\)](#), [Baker et al. \(forthcoming\)](#) and [Baker et al. \(2019\)](#).

<sup>2</sup>This pattern can be confirmed even in the aggregate consumptions. See [Figure 18](#) in the appendix [11](#). More

Figure 1: Dynamic Tax Elasticity of Taxable Spendings



Note: Reported in Baker et al. (forthcoming). Dashed vertical line indicates the timing of the consumption tax hike. It can be confirmed that household taxable spendings sharply rise one period before the tax hike, fall in the period of the tax hike, and gradually recover to the new steady state level afterwards.

the tax wedge is able to reproduce this dynamic pattern of taxable spendings and the tax wedge is the *only* investment friction that accounts for this pattern. The intuition of the slow recovery is simple. Since the households stock up the durables, they do not buy as much durables for some periods afterwards. In this way, the baseline model generates swift adjustment before the tax hike and gradual recovery after the tax hike. Surprisingly, a frictionless model (i.e., a model without the tax wedge) does not generate the slow convergence after the tax hike. Models with other investment adjustment costs, such as quadratic adjustment costs or fixed cost, also fail to reproduce the dynamic pattern. Thus, the tax wedge is the only investment friction that reproduces the dynamic pattern of the empirical tax elasticity. The model-implied tax elasticity of taxable spendings from my baseline model is 1.87 one period before the tax hike,  $-1.44$  in the period of the tax hike,  $-1.23$  in the following period, falling into the confidence interval of Baker et al. (forthcoming) without targeting them in calibration. The baseline model also generates the tax elasticity of durable spendings of 10.6, which Baker et al. (2019) estimate 8.1 – 12.8.<sup>3</sup>

generally, other stimulative policies to the durables also witness this slow recovery in the household spendings. For instance, Mian and Sufi (2012) confirms that CARS program induces the fewer automobile purchases for ten months after the expiration of the program.

<sup>3</sup>8.1 is from Column (3) in Table 2 and 12.8 comes from Table 3. As they discuss timing issue of purchase and registration, I sum up to 9.05 and 3.70 in Column (1), and obtain the value of 12.8.

Second, the tax wedge helps predict *true* stockpiling behavior. Without the tax wedge, the model allows the households to receive the consumption tax when they sell the durables. This unrealistic receipt of the consumption tax generates the following speculation opportunity over the dynamics. Suppose that an automobile is worth \$10,000 and the tax hike from 5% to 10% will take place tomorrow. The household buys the automobile and pays \$10,500 today. When selling the automobile with the tax rate of 10% next day, the household in the model without the tax wedge receives \$11,000. Thus,  $\$11,000 - \$10,500 = \$500$  minus the depreciation is a monetary profit of speculation by taking advantage of the tax hike in the model without the tax wedge. Knowing this speculation opportunity, the households excessively stockpile for the speculative motive but not for their own use. In this way, the model without the tax wedge generates strong stockpiling behavior based on the unrealistic mechanism. This misspecification problem influences the existing quantitative analysis. For instance, [Cashin and Unayama \(2016\)](#) estimate the intertemporal elasticity of substitution (IES) using the Japanese consumption tax hike. They construct a model of durables with the consumption tax but did not introduce the tax wedge. To reconcile with observed moderate stockpiling behavior using their model, they reach a conclusion that IES is low. But, their estimated value of IES must be biased downward due to the misspecification problem.

Next, incorporating life-cycle into the model is important to match the magnitude of the tax elasticities as the life-cycle motives largely dampen the elasticity. Recently, [Koby and Wolf \(2020\)](#) and [Winberry \(2020\)](#) show that (S,s) investment models generate infinitely high interest rate elasticity of investment. This high interest rate elasticity also emerges in a model of durables with (S,s) rules as [McKay and Wieland \(2019\)](#) study. As shown in [Correia et al. \(2008\)](#), the change in the interest rate has a similar implication on the household expenditures as the change in the consumption tax. Hence, resolution of this puzzle is a central problem this paper must address.<sup>4</sup> I find that incorporating the life-cycle structure makes the interest rate elasticity of the aggregate durable expenditure in the baseline model fit empirically reported values closely, without any additional frictions. This is because there is a mass of households who do not respond to changes in the interest rate (and consumption tax) in a life-cycle model. For instance, the young purchase the durable goods regardless of interest rate (and consumption tax) as they have very high marginal utility of durables. In contrast, the old do not buy durables whatever the current interest rate (and consumption tax) is, since they will die soon. In this way, the life-cycle motive dampens the model-generated elasticity. As discussed above, adding other adjustment costs (i.e., quadratic adjustment cost) to control the interest rate elasticity is not suitable here,

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<sup>4</sup>Intuitively, standard Euler equation in a model with the consumption tax,  $u(c) = \beta(1 + r')^{\frac{1+\tau^c}{1+(\tau^c)'}} u'(c')$  shows the connection between  $r$  and  $1 + \tau$ .

since the adjustment costs influence the dynamic pattern of the spendings, which makes the model behavior inconsistent with the empirical findings.

Heterogeneity matters for welfare implications of the consumption tax hike *over the transition* since the tax-induced inflation decreases the real value of preexisting wealth. Thus, wealthy households incur a higher burden of the tax hike than poor households. This mechanism works not only in the liquid asset but also in the durables. Thus, the presence of the durable widens the inequality of the welfare loss from the tax hike. These progressive effects do not exist when focusing on the stationary environment only. While the studies on the stationary environment may have contributed to establishing a common sense that consumption tax is regressive, this paper finds that the consumption tax hike is progressive on the transition dynamics.

Finally, I consider two counter-factual experiments. First, I consider an anticipated tax cut. Recently, Germany decreased the consumption tax rate temporarily during the on-going pandemic crisis since 2020 July. Since the model is highly non-linear due to the combination of the presence of the durables and the tax wedge, the prediction of the tax decrease is asymmetric to the tax hike. This finding suggests two implications. If the model is linearized, then the solution would not capture this asymmetry. Furthermore, when an econometrician runs a regression with the tax change experiment, the theory suggests that the empirical analysis should be separately implemented between the tax hike case and the tax cut case.

Second, I compare two tax hike schemes: a multi-times stepwise tax hike scheme and a one-time tax hike scheme. The multiple-times stepwise tax hikes scheme causes stockpiling before the first tax hike and the drop in the durable expenditure after the final tax hike. In reality, the economic slump is persistent and thus policy makers hope to avoid the drop in the durable expenditures in the midst of the recession. My result suggests that the policy makers can postpone the largest drop in the household expenditures by keeping raising the tax rate until the economic slump ends. Moreover, I find that the multi-times tax hikes scheme is better than the one-time tax hike scheme in terms of social welfare since the multi-times tax hikes scheme causes moderate change in price, which induces smoother adjustment.

The baseline model of this study is a two-asset life-cycle model with discrete choices. This class of the models is known to be computationally expensive to solve. I extend the latest endogenous grid method (hereafter EGM), Nested EGM, which is developed by [Druedahl \(2020\)](#) and solve the entire analysis, including the transition dynamics of the general equilibrium with my laptop. A novel feature of Nested EGM is by nesting the timing of the household's problem, we can divide the multi-dimensional optimization over the asset and durable into two steps of unidimensional problems. The algorithm applies the off-the-shelf EGM to this uni-dimensional part and solves the dynamic optimization problem fast.

### 1.1. Relation to Literature

To my best knowledge, this is the first paper to study the dynamic responses to the anticipated consumption tax changes with a general equilibrium model of consumer durables with the tax wedge. This paper aims to contribute to the three strands of literature mainly: unconventional fiscal policy, consumption tax reform, and the lumpy durable literature.

This paper contributes to the theoretical and quantitative understanding of the unconventional fiscal policy. [Correia et al. \(2013\)](#) theoretically prove that the unconventional fiscal policy resolves the limitation of conventional monetary policy in the zero lower bound using a representative agent New Keynesian model and show that it also achieves the efficient allocation. My model is an extension of [Correia et al. \(2013\)](#) with regard to the durables, the tax wedge, life-cycle, and heterogeneity. Once the durables are introduced into a model, responses to the preannounced consumption tax changes are significantly changed.

[Baker et al. \(forthcoming\)](#) also studies the unconventional fiscal policy both empirically and theoretically. They develop a consumer inventory model with shopping costs to account for the observed responses to the anticipated sales tax changes in the U.S. They focus on the U.S. sales tax system, i.e., local sales tax changes while I study changes in the nation-wide consumption tax rate. Therefore, I build an extended Bewley-Huggett-Aiyagari type model which is commonly used in the macroeconomic literature. Also, my model is able to reproduce the reported slow convergence to a new steady-state after the tax hike.

From the empirical side of the studies on the unconventional fiscal policy, [Baker et al. \(2019\)](#) empirically estimate the tax elasticity of automobile purchases in the U.S. [D’Acunto et al. \(2016, 2018\)](#) also study the effects of anticipated consumption tax changes and show that the readiness of the durable purchase increased before the consumption tax hike in Germany and Poland. To sum up, these empirical papers on consumption tax changes emphasized the roles of durable goods. This paper builds a bridge between these empirical findings and macroeconomic theory by introducing consumer durable goods into the theoretical model.

The second strand of the literature that I aim to contribute to is on consumption tax reforms.<sup>5</sup> Among them, [Altig et al. \(2001\)](#) and [Nishiyama and Smetters \(2005\)](#) studies a transition over the replacement of a realistic progressive income tax system with a flat consumption tax system, using a life-cycle model. They study the intergenerational and heterogeneous welfare costs of the introduction of the consumption tax. My paper revisits their analysis with a model of the durables.

My baseline model is the most similar to [Parodi \(2018, 2019\)](#) from the structural modeling

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<sup>5</sup>Krusell et al. (1997), Ventura (1999), Erosa and Ventura (2002), Correia (2010), Kitao (2011), Nakajima and Takahashi (2017a), Laczó and Rossi (2019) and Motta and Rossi (2019).

perspective. [Parodi \(2018\)](#) constructs a rich life-cycle heterogeneous agent partial equilibrium model with durable goods and the tax wedge to study the value-added tax system reform in Italy that changes from the current consumption tax system with reduced tax rates to a uniform tax rate. [Parodi \(2019\)](#) also uses a similar environment to study the optimal consumption tax rates for the Italian economy in the presence of reduced tax rates in a stationary environment. My paper differs from [Parodi \(2018, 2019\)](#) in that my focus is on the dynamic responses of the households to the anticipated tax change. I will argue that effects from the tax-induced inflation which causes heterogeneous welfare implication over the dynamics only, and therefore this paper complements her works by adding this dynamic perspective.

The third related literature is on the lumpy durable adjustments. There are a considerable amount of papers that contributed to this literature.<sup>6</sup> Among them, [Berger and Vavra \(2015\)](#) is a recent and particularly influential paper in the literature. They show that the response of the durable expenditure is more sluggish in recessions. [Berger and Vavra \(2015\)](#) also shows that this responsiveness is robust to the assumption of partial equilibrium or general equilibrium. My study applies the lumpiness to the responses to the anticipated consumption tax changes. Also, [Fernández-Villaverde and Krueger \(2007, 2011\)](#) and [Yang \(2009\)](#) study both empirically and theoretically the life-cycle profile of the durable expenditure. I find that life-cycle structure can dampen the interest rate elasticity of aggregate durable expenditure in a model and help match the observed elasticities.

Finally, my paper relates to the conventional monetary policy because a permanent consumption tax hike can be viewed as lowering the real interest rate in a single period from the household perspective. This equivalence between the fiscal and monetary policy is formally established in [Correia et al. \(2008\)](#). Thus, my work potentially contribute to the understanding intertemporal shifting due to the conventional monetary policy in a rich heterogeneous-agent model.

Many Heterogeneous Agent New Keynesian (HANK) models recently feature roles of durable, housing, or illiquid asset. For instance, [McKay and Wieland \(2019\)](#) and [Zorzi \(2020\)](#) study transmission mechanisms of monetary policy with a model of lumpy durable adjustments.

[Doepke and Schneider \(2006\)](#), [Sterk and Tenreyro \(2018\)](#), [Auclert \(2019\)](#) and [Doepke et al. \(2018\)](#) argue that inflation has a redistribution effect between the asset rich and the asset poor. [Nishiyama and Smetters \(2005\)](#) analyze the heterogeneous welfare cost of the tax-induced inflation on the consumption tax context. I follow and revisit those insights with a model of the durables.

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<sup>6</sup>For instance, [Lam \(1989, 1991\)](#), [Grossman and Laroque \(1990\)](#), [Caballero \(1990, 1993\)](#), [Eberly \(1994\)](#), [Leahy and Zeira \(2005\)](#), [Luengo-Prado \(2006\)](#), [Favilukis et al. \(2017\)](#), [Guerrieri and Lorenzoni \(2017\)](#), [Gavazza and Lanteri \(2018\)](#).



## 1.2. Layout

In what follows, I explain the baseline model in the next section. Section 3 overviews the computational solution. Section 4 discusses parametrization. Section 5 illustrates the basic result in a stationary equilibrium. From section 6, I discuss the results of the consumption tax change. The section 6 discusses the long-run effects of the consumption tax change. Section 7 is a main section of this paper that focuses on the dynamic responses to the anticipated consumption changes, assuming the partial equilibrium. Section 8 solves the general equilibrium dynamics and confirms that equilibrium assumption does not affect the quantitative results significantly.

## 2. Baseline Model

### 2.1. Households

Time is discrete. Let  $t = 0, 1, \dots$  be time and  $j$  be age. Households live for finite periods  $J$ . They start working as soon as they are born and continue working until period  $J^R - 1 < J$ . After the retirement age  $J^R$ , they receive social security  $ss$ . The households have no bequest motives and are born with neither asset  $a$  nor durable stocks  $d$ . There exists bequest in the form of the durable despite the deterministic lifetime and no bequest motive because some households own the durables in the last period of the life. The sum of the durables that deceased households own equals the total bequest, that is  $D_J = B$ . This total bequest  $B$  is equally distributed to all the alive households at the end of every period. Let  $b$  denote the bequest each household receives. For notational simplicity, I omit time and age subscript if not necessary.

The households supply labor inelastically. In beginning of every period, the working households draw an idiosyncratic labor productivity from a Markov process  $\pi(e'|e)$  and receive labor earnings  $w\kappa_j e$  where  $\{\kappa_j\}_{j=1}^{J^R-1}$  is a deterministic age-dependent component of the labor earnings. This  $\kappa_j$  is introduced to account for the hump-shaped labor earnings. An initial productivity  $e_0$  at the age of 0 is assumed to be drawn from the ergodic distribution of the Markov process. For notational simplicity, let  $y_j(e)$  denote the labor earnings and social security. That is,

$$y_j(e) = \begin{cases} w\kappa_j e & \text{if } j < J^R \\ ss & \text{if } j \geq J^R. \end{cases}$$

Also, the households receive an interest income  $(1 + r_t)a_{-1}$ , and spend the sum of labor and interest income and the bequest on either non-durables  $c \geq 0$  or durable purchases  $x^d$ , or save in



liquid asset  $a$ . The durables depreciate at the rate of  $\delta^d$ . When purchasing the non-durables and the durables, the households face the same consumption tax rate  $\tau^c$ . The flat consumption tax rate is assumed because the consumption tax rate is uniform across goods and across regions in Japan during sample periods I study.<sup>7</sup> The households cannot borrow, i.e.  $a \geq 0$  is imposed.

Furthermore, the households are subject to a friction for the durable adjustments. The households face the tax wedge which reflects the realistic consumption tax system. The consumption tax is charged only when the seller is business. That is, when the households sell their own durable goods to the market, this transaction is not subject to the consumption tax. The tax wedge takes the following form:

$$\mathcal{T}(d, d_{-1}) = \begin{cases} (1 + \tau^c)x^d & \text{if } x^d \geq 0 \\ qx^d & \text{if } x^d < 0, \end{cases}$$

where  $x^d = d - (1 - \delta^d)d_{-1}$ ,

where  $q$  is an indicator whether the tax wedge in a model is turned on. I set  $q = 1$  in a benchmark case. I refer to frictionless or a symmetric tax case when  $q = (1 + \tau^c)$  in which the tax wedge is turned off. In this manner, the households in the baseline model face different consumer prices between buying and selling the durables.

To sum up, the households face the budget constraint below,

$$(1 + \tau^c)c + a + \mathcal{T}(d, d_{-1}) = (1 + r)a_{-1} + y_j(e) + b. \quad (1)$$

and non-negativity constraints  $(a, c, d) \geq 0$ . The household state space is  $(j, a_{-1}, d_{-1}, e)$ . Let  $\mu_j(a_{-1}, d_{-1}, e)$  be the distribution of the households at the age of  $j$ . Note that without the tax wedge (i.e. setting  $q = 1 + \tau^c$ ), the households receive the consumption tax  $\tau^c x^d$  when selling the durables ( $x^d < 0$ ).<sup>8</sup> I will discuss the importance of this misspecification later in detail in section

7.2.

The households' dynamic optimization problem can be written as

$$v_j(a_{-1}, d_{-1}, e) = \max_{(a, c, d) \geq 0} u(c, d) + \beta \sum_{e'} v_{j+1}(a, d, e') \pi(e' | e) \quad (2)$$

s.t. (1)

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<sup>7</sup>As of 2019 October, Government of Japan introduced a reduced tax system.

<sup>8</sup>That is, suppose  $q = 1 + \tau^c$ . Then, when  $x^d < 0$ , the budget constraint implies that  $(1 + \tau^c)c + a = (1 + r)a_{-1} + y_j - \underbrace{(1 + \tau^c)x^d}_{>0}$ .

where  $v_{j+1}(a, d, e') = 0$  for all  $(a, d, e')$ .

Alternatively, it seems to be more intuitive to rewrite the optimization problem explicitly with discrete choices in the following way. The households choose either upward adjusting the durables (i.e., buy the durables), inaction, or downward adjusting the durables (i.e., sell the durables),

$$v_j(a_{-1}, d_{-1}, e) = \max\{v_j^{up}(a_{-1}, d_{-1}, e), v_j^{inact}(a_{-1}, d_{-1}, e), v_j^{down}(a_{-1}, d_{-1}, e)\}.$$

Each choice-conditional problem is solved separately in the following manner.

When upward adjusting the durables, the households solve

$$\begin{aligned} v_j^{up}(a_{-1}, d_{-1}, e) &= \max_{(a, c, d) \geq 0} u(c, d) + \beta \mathbb{E}[v_{j+1}(a, d, e') | e] \\ &\text{s.t. } (1 + \tau^c)[c + x^d] + a \\ &= (1 + r)a_{-1} + y_j(e) + b \\ &x^d > 0. \end{aligned}$$

Notice that  $x^d > 0$  is explicitly imposed.

When choosing the inaction, the households solve

$$\begin{aligned} v_j^{inact}(a_{-1}, d_{-1}, e) &= \max_{(a, c) \geq 0} u(c, (1 - \delta^d)d_{-1}) + \beta \mathbb{E}[v_{j+1}(a, (1 - \delta^d)d_{-1}, e') | e] \\ &\text{s.t. } (1 + \tau^c)c + a \\ &= (1 + r)a_{-1} + y_j(e) + b. \end{aligned}$$

The households who choose inaction simply carry the depreciated durable  $(1 - \delta)d_{-1}$  to the next period. Thus, this optimization is essentially one-dimensional as in standard incomplete market models. This becomes key in computation. This inaction problems allows us to apply the EGM technique for this choice-specific Bellman equation with special treatment for the kinky value functions due to the discrete choices. The Nested EGM heavily takes advantage of this structure and I will discuss the detail in appendix [10.1](#).

Finally when downward adjusting the durables, the households solve

$$\begin{aligned}
v_j^{down}(a_{-1}, d_{-1}, e) &= \max_{(a, c, d) \geq 0} u(c, d) + \beta \mathbb{E}[v_{j+1}(a, d, e') | e] \\
&\text{s.t. } (1 + \tau^c)c + qx^d + a \\
&\quad = (1 + r)a_{-1} + y_j(e) + b \\
&\quad x^d < 0.
\end{aligned}$$

Again, notice that  $x^d < 0$  is imposed.

## 2.2. Government

The government collects the consumption tax as a revenue and use it to finance total pension  $SS = \sum_{j \geq J^R} \int ss d\mu_j$  and government expenditure  $G$ .

$$G + SS = \tau^c(C + X_+^d) \quad (3)$$

where  $C = \sum_j \int cd\mu_j$  is an aggregate non-durable consumption and  $X_+^d = \sum_j \int_{x^d > 0} x^d d\mu_j$  is a sum of positive aggregate expenditures on the durables because consumption tax is paid only when the durable expenditure is positive.

In my model, additional tax revenue from the tax hike is spent on the government expenditure  $G$ . The government expenditure  $G$  is not used anywhere and wasteful. An alternative modeling of the use of the tax revenue is to transfer it to the households or mix of the government expenditure and the transfer. I choose the government expenditure as a benchmark for three reasons. First, I want to isolate the effect of anticipated consumption tax hike from effects of the transfer. It is well known that Hand-to-Mouth (including wealthy Hand-to-Mouth) households strongly react to the lump-sum transfer in two asset models as [Kaplan and Violante \(2014\)](#) point out. Therefore, the lump-sum transfer potentially boosts the stockpiling behavior in response to anticipated consumption tax change in a non-trivial way. This makes interpretation of the household responses complicated and we cannot identify whether the heterogeneous response is due to the tax hike or the transfer. Also, if the government transfers the revenue to the households, the model would generate stronger stockpiling responses. In other words, my result can be seen as lower bound in this respect. Second, this modeling is intended to reflect the fact that the additional tax revenue is mostly for the sustainability of the Japanese Government Debt and used for the repayment of the debt. While the accumulated government debt is unmodeled, the government expenditure can be interpreted as expenditures on such a unmodeled factor.<sup>9</sup> Third, this

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<sup>9</sup>Other unmodeled objects include frequent natural disasters (e.g. typhoons, earthquakes) and spending on the

modeling is computationally easy. Given the OLG structure with two assets and discrete choices, burden of the computation, especially for the dynamics, is non-negligible.

### 2.3. Firm

The firm side is standard. I assume a representative competitive firm which owns Cobb-Douglas technology for the final output. This firm follows

$$\begin{aligned} r + \delta &= \alpha K^{\alpha-1} N^{1-\alpha} \\ w &= (1 - \alpha) K^{\alpha} N^{-\alpha}, \end{aligned} \tag{4}$$

where  $K$  and  $N$  are aggregate capital and labor respectively. This output is used for the non-durable and the durable consumption, the capital investment and the government expenditure.

### 2.4. Dealers

It is worth noting that the households always pay the consumption tax when they buy the durable goods. This implicitly means that households are not allowed to trade durables between households in a model. In reality, the direct and private trades between households are a way to avoid the consumption tax. If such an opportunity exists in a model, the households choose it and avoid the consumption tax.

I assume that the competitive dealers manage all durable transactions. That is, the dealers purchase the durables from both the firm and the households, and sell the durable goods to the households. When the households sell the durables to the dealer, the households receive  $q$  per unit of the durables. Similarly, when the households buy the durables from the dealers, they pay 1 per unit to the dealers and  $\tau^c$  per unit to the government.<sup>10</sup> The used and new durables are assumed to be perfect substitute. Therefore, the households do not distinguish the new and the used durables.

### 2.5. A Recursive Competitive Equilibrium

A recursive competitive equilibrium of this model consists of a pair of functions,

$$\left( \{v_j, g_j^d, g_j^a\}_j, \{\mu_j\}, C, D, K, N, w, r \right)$$

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on-going pandemic crisis.

<sup>10</sup>Since I assume perfect pass-through of the consumption tax, the households incur full incidents of the consumption tax.

that solves the households and the firm optimization problems and clear markets for asset, labor and output. That is, the equilibrium satisfies

- (i)  $v_j$  solves age  $j$  household problem (2) given prices  $(w, r)$ .  $g_j^d$  and  $g_j^a$  are policy functions for the durables and the assets respectively.
- (ii) The firm solves the static optimization and follows (4).
- (iii) The government observes its budget constraint (3).
- (iv) The distribution  $\{\mu_j\}$  follows

$$\mu_{j+1}(s, e') = \int_{\{(a_{-1}, d_{-1}, e) | (g_j^d(a_{-1}, d_{-1}, e), g_j^a(a_{-1}, d_{-1}, e)) \in s\}} \pi(e' | e) d\mu_j(a_{-1}, d_{-1}, e).$$

- (v) The labor market clears:

$$N = \sum_{j=1}^{J^R-1} \int e \kappa_j d\mu_j.$$

- (vi) The asset market clears:

$$K = \sum_{j=1}^J \int a_{-1} d\mu_j.$$

- (vii) The good market clears:

$$C + D + G + K' = K^\alpha N^{1-\alpha} + (1 - \delta)K + (1 - \delta^d)D_{-1},$$

where  $D_{-1} = \sum_{j=1}^J \int d_{-1} d\mu_j$  is an aggregate durable stock.

### 3. Computation

Solving this class of the models is computationally demanding in the presence of the discrete choices and a multidimensional optimization. The discrete choices cause non-concave regions on the both unconditional and choice-specific value functions and therefore derivative based optimization methods become less accurate or unstable. Researchers may need to worry the local optima. Moreover, the multidimensionality in the optimization often suffers the curse of the dimensionality. When solving perfect foresight dynamics with a simulation period of  $T$ , we must

solve  $J + T - 1$  generations' dynamic optimization problems. Hence, efficient solution method for the dynamic programming is necessary.

Drue Dahl (2020) develops a novel algorithm to solve this class of models with multidimensional optimization problem with the discrete choices. I applied his Nested EGM to the model with the tax wedge and show that this algorithm gives a well-behaved solution to the richer model than the benchmark model he used for exposition. While details of the algorithm are explained in the appendix 10.1, this section briefly overviews ideas of Nested EGM.

The critical insight of the Nest EGM is nesting a model to reduce a dimension of choice-conditional optimization problems, introducing the additional timing structure within a period. That is, the Nest EGM views both upward and downward adjusting problem as sequential. The households are assumed to, first, decide how much durables they buy, and then decide how much non-durables. Importantly, this nesting timing assumption allows us to use the consumption function derived by the inaction problem for the both adjusting problems to reduce a dimension of the optimization problems. Thanks to this dimension reduction technique, the resulting optimization problems for both adjusting become one dimensional. These maximization problems are solved with standard VFI.<sup>11</sup>

Since the dimension of the optimization for the inaction problem is essentially one in computation, we can use EGM with upper envelope algorithm, developed by Drue Dahl and Jørgensen (2017) and Iskhakov et al. (2017). We extend these techniques for the model with the tax wedge or the partial irreversibility constraint.<sup>12</sup>

I use this algorithm for solving both the stationary equilibrium and the perfect foresight equilibrium path.

## 4. Parameterization

This section discusses the parametrization of the models. The model frequency is annual. consumption tax rate  $\tau^c$  is set 0.05 and 0.08 in the baseline which were actual consumption tax rates in Japan until 2014 April and since then until 2019 October.

I choose Japan as an application of my analysis for two reasons. First, there was no reduced tax system in Japanese consumption tax system with few exceptions until 2019 October.<sup>13</sup> This

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<sup>11</sup>Due to the discrete choice and a kinky value function, this optimization problem may have many local maxima. I use Rowan's Subplex algorithm (a local optimization method) provided by NLOpt toolbox on very finely-spaced subintervals and find a local maximum in the subintervals. And then I take the maximum from these local maxima in practice.

<sup>12</sup>Also, I solved a model with fixed-cost and quadratic adjustment cost with this solution method.

<sup>13</sup>The exceptional transactions are medical expenses, tuition fees, funeral fees, and rent.

consumption tax system allows to keep the analysis simple and transparent. Second and more importantly, the Japanese consumption tax hike from 5% to 8% was full pass-through to consumer prices because the government prohibited the tax change related price changes.<sup>14</sup> Therefore, I focus on the effect of the consumption tax on the households.

The Cobb-Douglas utility function is commonly assumed in the durable literature, following Ogaki and Reinhart (1998). However, the Cobb-Douglas utility specification in a simple life-cycle model with the consumer durables does not reproduce empirically observed hump-shape in durable expenditure as Fernández-Villaverde and Krueger (2011) study. Thus, I adopt non-homothetic utility function below as with Parodi (2019),

$$u(c, d) = \frac{(c^\theta (d + \epsilon_d)^{1-\theta})^{1-\sigma}}{1-\sigma}$$

where  $\epsilon_d > 0$  and  $\epsilon_d$  is not close to 0. This Stone-Geary functional form implies that the durable stocks are *luxury*, consistent to the finding in Pakoš (2011). Typically, since the households in a life-cycle model get rich in middle-age in terms of both asset and earnings, the middle-age households increase the durable purchase. In this manner, this utility functional form helps reproduce the hump-shaped durable expenditure. I pin down  $\theta$  and  $\epsilon_d$  to simultaneously match the ratio between initial level and peak of average durable expenditure, and aggregate non-durable and durable share. Then, I set  $\sigma = 2.0$  and  $\beta = 0.977$  which are standard values in the literature.<sup>15</sup>

Next, there is no official data for the durable depreciation in Japan. Thus, I take a depreciation rate of each durable goods (e.g. furniture, appliances, automobiles) from Fraumeni (1997), and compute the weight average of the depreciation rates using the share of the expenditure from National Survey of Family Income and Expenditure.

The tax wedge parameter  $q$  is set to be 1.0 in a benchmark case. But, to examine the role of the tax wedge, I compare the baseline model with the model without the tax wedge in which I set  $q = (1 + \tau^c)$ . Also,  $q < 1.0$  has been interpreted as a resale adjustment cost in the investment and durable literature and has been extensively studied.<sup>16</sup> Thus, I discuss the alternative value case in the appendix 13.2. The choice of  $q$  affects the wealth formation in the stationary equilibrium because the durables play a role as an insurance if  $q$  is high. Hence, the higher the  $q$  is, the more households save in durables in the stationary equilibrium. But, I confirm that the stockpiling behavior over the transition is almost irrelevant to choice of  $q$ .

As for the calibration of idiosyncratic shocks, I assume the simplest AR(1) labor productivity

<sup>14</sup>See the following link for the detail. [https://www.mof.go.jp/tax\\_policy/summary/consumption/250910tenka.htm](https://www.mof.go.jp/tax_policy/summary/consumption/250910tenka.htm)

<sup>15</sup>High discount factor  $\beta$  is common in Japanese economy literature, for example, Hayashi and Prescott (2002) and Yamada (2012).

<sup>16</sup>For the literature of the lumpy durables with the partial irreversibility constraint, see Lam (1989) for instance.



Table 1: Parameters in the Baseline Model

parameters	values	description
$\beta$	0.977	discount factor
$\sigma$	2.0	risk aversion
$\delta$	0.1	annual depreciation rate of the capital
$\delta^d$	0.15	annual depreciation rate of the durables
$\epsilon_d$	1.47	Stone-Geary preference parameter
$\theta$	0.77	utility share of the non-durables
$ss$	0.64	social security
$q$	1.0	resale adjustment friction
$\rho$	0.9	persistence of the idiosyncratic shocks
$\sigma_\epsilon$	0.2072	std of the idiosyncratic shocks

process.

$$\ln e' = \rho \ln e + \epsilon'.$$

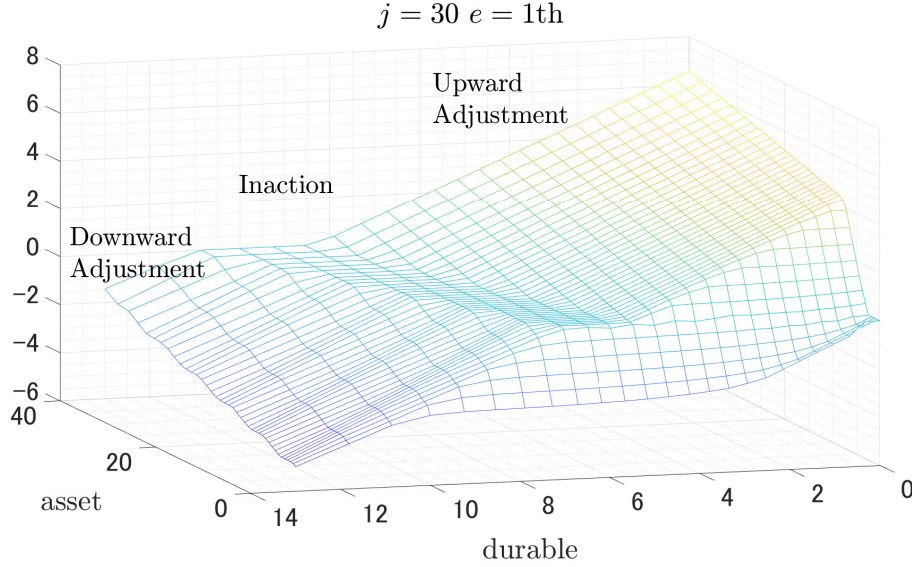
I follow a calibration strategy of [Nakajima and Takahashi \(2017b\)](#) for the parameters of the idiosyncratic shocks. There is no suitable panel data for the estimation of the earnings in Japan. Thus, they set persistence  $\rho$  of the shock to be standard value of 0.9 a priori.<sup>17</sup> Then, the age-dependent components  $\{\kappa_j\}_j$  is set so that labor earnings in the model match observed hump-shape in average labor earning reported in [Yamada \(2012\)](#). I make it annual by interpolating with the cubic spline. [Lise et al. \(2014\)](#) combines four major data in Japan to estimate volatility of shocks: Basic Survey on Wage Structure (BSWS), the Family Income and Expenditure Survey (FIES), the National Survey of Family Income and Expenditure (NSFIE), and the Japanese Panel Survey of Consumers (JPSC). To be consistent with [Lise et al. \(2014\)](#), the earnings dispersion conditional on  $\{\kappa_j\}$  is generated by idiosyncratic shocks and then I set  $\sigma_\epsilon = 0.2072$ . Finally I use the [Rouwenhorst \(1995\)](#) method to discretize this AR(1) process with 13 grid points. The social security is set at  $ss = 0.64$  following the report of the [OECD \(2017\)](#), which includes voluntary components.

## 5. Stationary Equilibrium

Figure 2 is a policy function of the durable expenditure  $x^d = g_j^d(a_{-1}, d_{-1}, e) - (1 - \delta^d)d_{-1}$ . Since the tax wedge generates a non-linearity in the budget constraint, the durable expenditure

<sup>17</sup>For example, [Floden and Lindé \(2001\)](#) finds 0.914 for the U.S. [Alonso-Ortiz and Rogerson \(2010\)](#) report 0.94 for the US.

Figure 2: Durable Expenditure Function  $x^d = g_j^d(a_{-1}, d_{-1}, e) - (1 - \delta^d)d_{-1}$ .



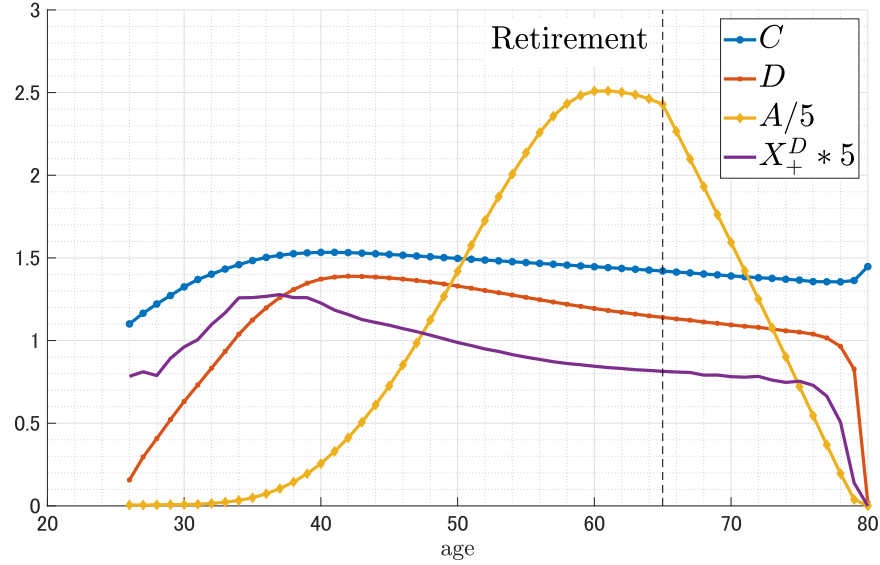
*Note:* The policy function of the durable expenditure shown is that of age of 55 ( $j = 30$ ) with the lowest idiosyncratic shock realization. The durable expenditure function exhibits high non-linearity due to the tax wedge.

function exhibits high non-linearity. The durable expenditure function consists of three regions: upward adjustment, inaction, and downward adjustment. Movement across different regions means the extensive margin.

Next, I assume the population of each generation is 10,000 in the model economy so that total population is large enough ( $J \times 10,000 = 550,000$ ) and run the stochastic simulation. Figure 3 shows the average of non-durable consumptions, durable stocks, assets, and positive durable expenditure profiles over the lifetime. The asset profile is much taller than other two profiles and thus scaled down while the durable expenditure profile is also scaled up. Due to the extensive margin the individual profiles show some spikes over the life-cycle but the spikes in individual level are canceled out in aggregation. Therefore, the Figure 3 looks smooth despite the presence of the discrete choice.

The Figure 3 shows that the average of the households increase the non-durable  $c$  and durable consumptions  $d$  in youth until late 30s or early 40s. The growth of the durable consumptions  $d$  is more persistent than non-durables  $c$  because of the Stone-Geary utility function. Since the middle-age households become richer in both earnings and assets, the middle-age households buy more luxurious durables than the young. Due to this effect, the baseline model replicates the hump-shaped positive durable expenditure  $x^d$  profile. After the age of 40, the average of both consumptions continue declining steadily toward the near end of life. At the end of the life, the

Figure 3: Average Life-cycle Profiles: the Non-Durables, the Durables Stock and Asset



*Note:* Average life-cycle is obtained from stochastic simulation. Each generation consists of 10,000 households. The durable expenditure profile exhibits hump-shape thanks to the Stone-Geary utility function.

dying households substitute the durables with the non-durables and increase the non-durable consumption but decrease durables expenditure. The average households start accumulating the liquid asset  $a$  around the age of 35. In their middle-age, the liquid assets  $a$  sharply rise and hit a peak before the retirement age. After the retirement the households decumulate the assets since social security is less than the average of the labor earnings.

These life-cycle profiles turn out to be important for short-run responses to the anticipated consumption tax hike. I will show that while the young do not exhibit the strong stockpiling of the durables in response to the anticipated hike, all the other generation hoard the durable well. I will discuss this again later in the subsection 7.2.4.

Also, it is worth reporting the wealth distribution in my baseline model and from the observation. The wealth distribution I refer to is reported by Kitao and Yamada (2019). They use the National Survey of Family Income and Expenditure (NSFIE). The NSFIE is a cross-sectional data which has a very large sample size with 55,000 to 60,000 households in each survey year without top-coding. The survey is collected every five year since 1959. This paper compares with data in 2014 because it is the latest one in Kitao and Yamada (2019). In particular, I compare wealth share from their report with the asset share in my model economy for three reasons. First, their definition of the household wealth is the sum of financial assets and thus does not include either

Table 2: Wealth Share owned by each quintile

	1st	2nd	3rd	4th	5th	top10%	top1%
Data (2014)	0.3%	3.7%	9.8%	21.3%	64.9%	45%	10.2%
Baseline	0.1%	2.4%	12.6%	27.3%	57.6%	35.8%	5.7%

*Note:* Data (2014) is the figure reported in [Kitao and Yamada \(2019\)](#). The 1st quintile, for example, means the bottom of the distribution.

real assets or durables. Second, they also exclude debt associated with purchases of real estate. In my model economy, the borrowing is strictly prohibited, and therefore the net assets coincide with wealth. Third, they report the share of the wealth but not that of net wealth.

Table 2 is the wealth share both in the model and the NSFIE. Overall, the distribution generated from the baseline model matches the observed wealth distribution well. It is noteworthy that Japanese wealth share is much less concentrated than that of the U.S. Since common difficulty is to reproduce the fat right tail of the wealth distribution, our model also misses right tail of the distribution.<sup>18</sup> Also note that my baseline model does not incorporate adjustment frictions typically used to match the wealth distribution such as heterogeneous discount factor or idiosyncratic shocks to the return. Overall, the baseline model matches the wealth share.

## 6. Long-Run Effects of the Consumption Tax Change

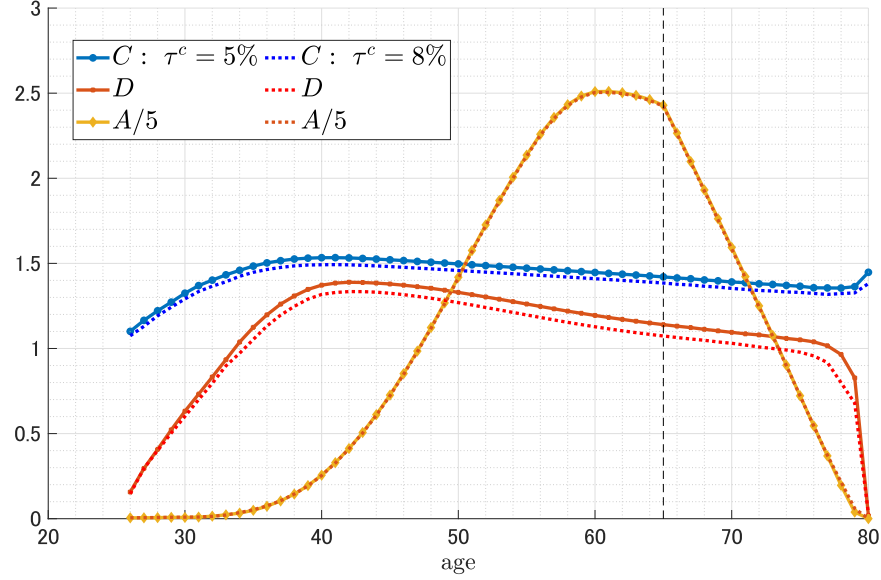
This section studies influences of the consumption tax hike on life-cycle consumption profiles and distribution in the long-run. Specifically, I will compare two stationary equilibria under the two consumption tax rate, 5% and 8%. 5% consumption tax rate was the Japanese consumption tax rate until 2014 March, and 8% was between 2014 April and 2019 September.

### 6.1. Intergenerational Effects

In figure 4, solid lines are the profiles with the consumption tax rate of 5% and dashed lines are the profiles with  $\tau^c = 8\%$ . The figure demonstrates both the non-durable and durable consumptions profile are shifted downward almost in parallel. These observations imply that burdens of the consumption tax on the consumptions are borne by all the generations fairly equally in the long-run. Also, as known in the literature, consumption tax change in the long-run do not affect asset accumulation in my baseline model as well. This is because the intertemporal relative price

<sup>18</sup>For the survey of the wealth distribution literature, for example, see [De Nardi and Fella \(2017\)](#) and [Benhabib and Bisin \(2018\)](#).

Figure 4: Life-cycle Consumptions Profile in response to Consumption Tax Hike.



Note: Solid lines are when  $\tau^c = 5\%$  and dotted lines are when  $\tau^c = 8\%$ . The tax hike influences the composition of consumptions disproportionately since the durable is luxury.

remains constant in the stationary equilibrium.

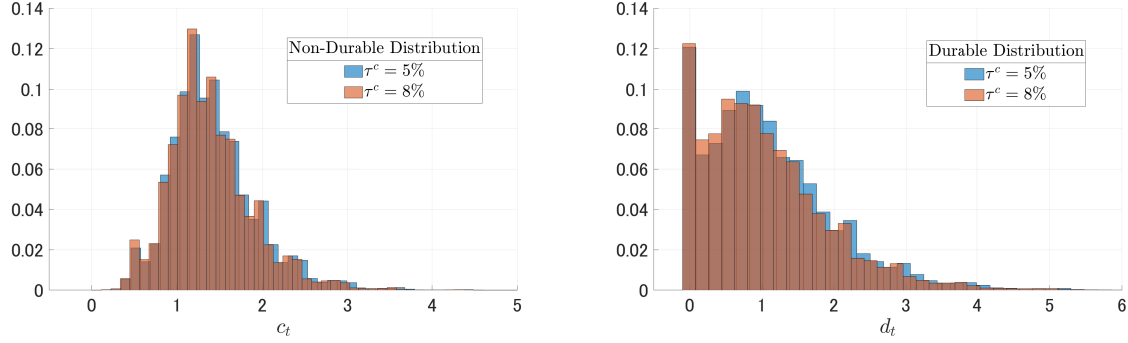
## 6.2. Cross-sectional Effects

Next, I am interested in the cross-sectional effects in this model. In the literature, it has been controversial whether or not fundamental tax reform which replaces current complex income tax with a flat consumption tax induces exacerbate inequality.<sup>19</sup> Regarding the tax reform, there are two policy changes: (i) an increase in consumption tax rate and (ii) elimination of the income tax. This section focuses on the former effect with a model of the durables.

Figure 5 shows distributional changes of consumptions against the consumption tax hike in the long-run. A left panel shows the changes in the non-durable distribution and a right panel shows that in the durable distribution. Both distributions see the larger impacts in the right than left, and thus result in being more right-skewed. This is because the poor and have stronger incentive to maintain the level of consumptions than the rich. The resulting right-skewness has a potential policy implication that the consumption tax affects the rich more than the poor in the long-run. In fact, standard deviation of non-durables and durables decrease by 2.81% and 2.71% respectively, implying that the increase in the consumption tax decreases the consumption

<sup>19</sup>For instance, Ventura (1999) and Altig et al. (2001) argue the tax reform causes further inequality in earning and wealth and the poor lose with the flat tax, whereas Correia (2010) argue opposite.

Figure 5: Changes in the Stationary Distributions



Note: Left Non-Durable distribution, Right Durable distribution.

inequality. Regarding the change in asset distribution, there was no change and thus I do not display it here.

## 7. Short-Run Impacts: Partial Equilibrium

The focus of this paper is on the study of stockpiling behavior in response to the anticipated consumption tax change. The analysis of the stockpiling in response to the anticipated consumption tax changes is implemented by solving the perfect foresight path. First, I test whether the baseline model can replicate *dynamic pattern of empirical tax elasticities*.

I start the analysis with the assumption of the partial equilibrium for the perfect foresight path in this section since the partial equilibrium dynamics is a simpler than general equilibrium and thus is interpretable more clearly. The economy faces constant prices  $(r, w)$  over the sample periods and this price path  $(r, w)$  are those at the initial stationary equilibrium. I show the results from the general equilibrium in the next section.

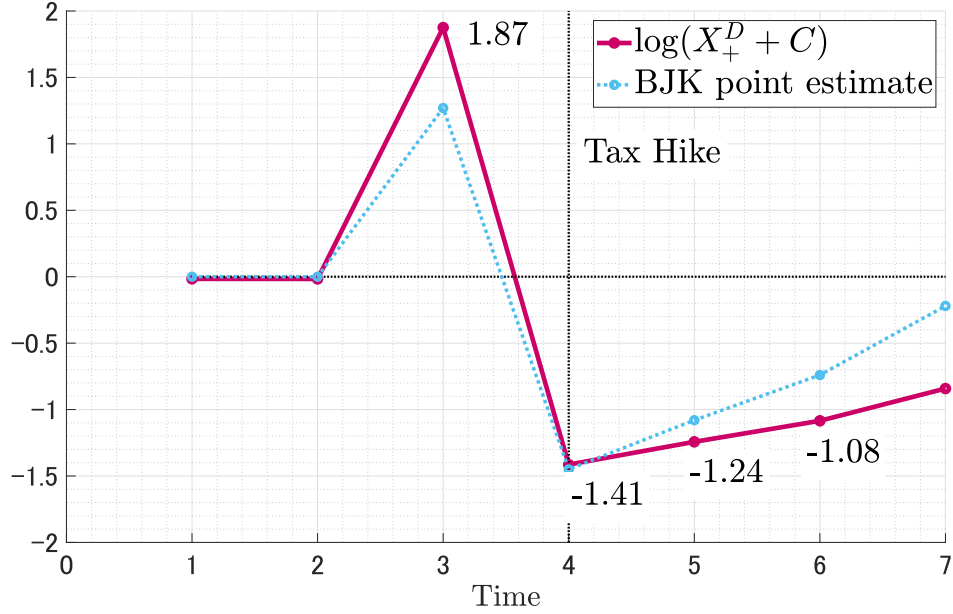
### 7.1. Tax Elasticities

#### 7.1.1. Taxable Spendings

Figure 6 compares the elasticity of taxable spendings  $(X_+^D + C)$  from the baseline model and the empirical result reported in Baker et al. (forthcoming). I assume that tax hike is announced before the  $t = 1$  and implemented  $t = 4$  in the model.<sup>20</sup> Since their data frequency is monthly and

<sup>20</sup>As will be clear in the next subsection, the announcement of the tax hike has a negative income effect. But we do not see the effects in the Baker et al. (forthcoming) result because local governments had different lags between announcement and implementation of the tax change in their sample. In this way, empirical result mutes the negative

Figure 6: Tax Elasticity of Taxable Spendings ( $X_+^D + C$ )



Note: BJK refers to the point estimate in Figure 1 also from Baker et al. (forthcoming). Numbers in the figure indicate the model-generated tax elasticity in every period. The baseline model generates the empirically plausible dynamic pattern of the taxable spendings.

my model frequency is annual, I do back-of-envelope calculation for the frequency adjustment.<sup>21</sup> Figure 6 shows that the baseline model replicates both dynamic pattern of tax elasticity and levels of these values.

First, the baseline model replicates the dynamic pattern of the empirical elasticity. Key dynamic patterns of the empirical tax elasticity are two fold. First, the empirical tax elasticity spikes up only in period 3 but not in period 1 and 2. Second, the empirical elasticity plummets in period 4 and slowly recover to the new stationary equilibrium level. The baseline model exactly replicates dynamic pattern.

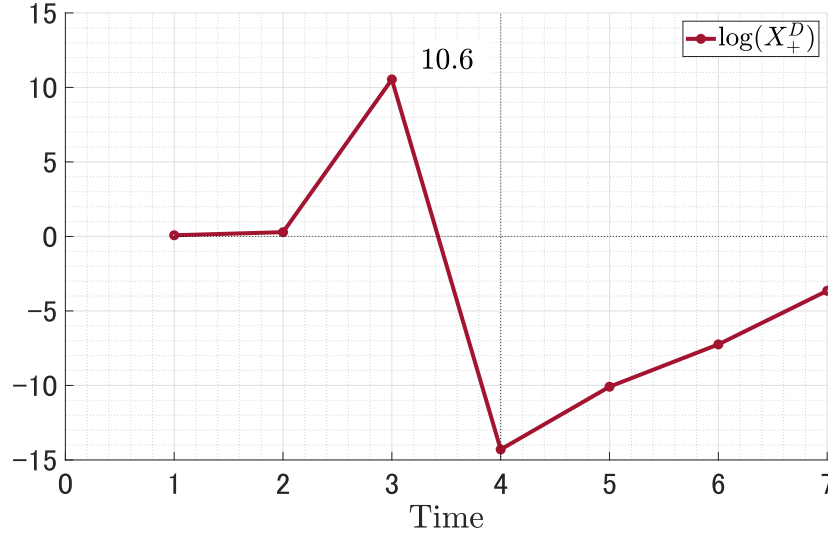
Second, the baseline model is roughly consistent to the reported point estimate and all the values of the model-implied elasticity falls within empirical 95% confidence interval. It is noteworthy that these moments in the model are not targeted in calibration. The model sees some discrepancies in the elasticity. A potential reason for this discrepancy is compositions in sample. Since Baker et al. (forthcoming) uses Nielsen which mainly collects data on groceries and sundries, their

income effects. By assuming the announcement before  $t = 1$ , I also mute the negative income effect in the simulation period shown.

<sup>21</sup>First, I increase the tax rate by 12% in the annual model that corresponds to 1% increase in a monthly basis. I compute the elasticity  $\epsilon_{X_+^D+C|12\%}$  of 12% tax hike and then obtain  $\epsilon_{X_+^D+C|1\%} = \epsilon_{X_+^D+C|12\%}/12$ .



Figure 7: Tax Elasticity of Durable Expenditure ( $X_+^D$ ) from the baseline model.



Note: Baker et al. (2019) report that the tax elasticity of automobile purchases one period before the tax hike is estimated between 8.1 – 12.8. The baseline model predicts the elasticity of 10.6 one period before the tax hike.

sample set do not contain many of typical consumer durables such as autos, appliances, furniture etc. Thus, the share of the durables in their sample must be smaller than aggregate level share, which dampens the elasticity. Since my model is calibrated to the macroeconomic level data, it is natural that my model predicts higher elasticity than the result in Baker et al. (forthcoming).

Overall, my model replicates the dynamic pattern of the empirical tax elasticity. This is the strong evidence that the baseline model is correctly specified for the consumption tax change.

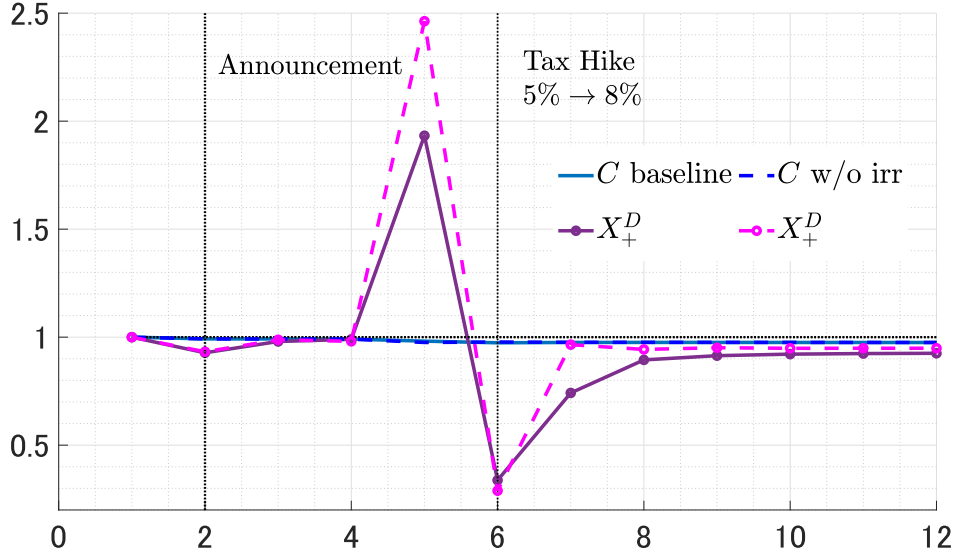
#### 7.1.2. Durable Expenditure

Next, I compare the tax elasticity of the aggregate durable expenditure with the result from Baker et al. (2019). They estimate the tax elasticity of the auto sales and obtain the value of 8.1 – 12.8 one month before the tax hike.<sup>22</sup>

Figure 7 shows the elasticity of the aggregate durable expenditure from the baseline model. I did same back-of-envelope calculation as the previous subsection 7.1.1. I find the elasticity one period before the tax hike from the baseline model is 10.6, which again is consistent to the empirical finding in Baker et al. (2019).

<sup>22</sup>8.1 is from Column (3) in Table 2 and 12.8 comes from Table 3. As they discuss timing issue of purchase and registration, I sum up to 9.05 and 3.70 in Column (1), and obtain the value of 12.8.

Figure 8: Consumption Tax Hike 5%  $\rightarrow$  8%



*Note:* The economy start at stationary equilibrium at  $t = 1$ . In beginning of  $t = 2$ , the government announces that it will increase the tax in  $t = 6$ . Solid lines depict baseline result and dashed lines show results of a model without the partial irreversibility constraint. The values are normalized by the initial stationary equilibrium values.

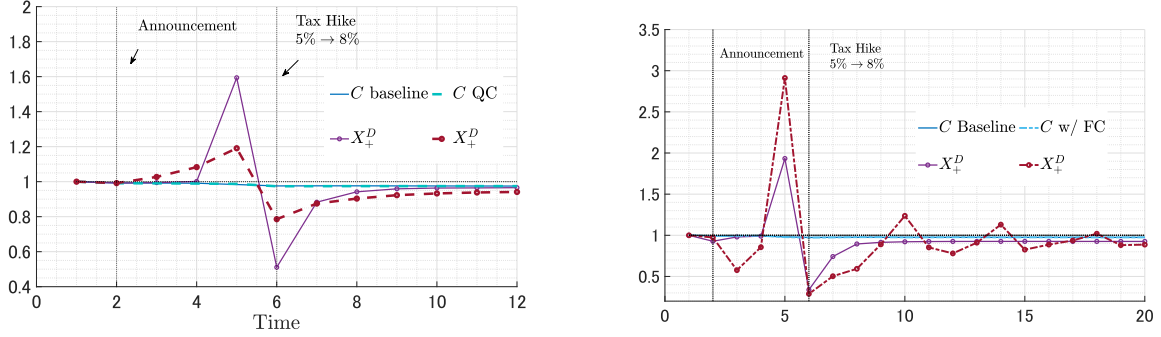
## 7.2. Baseline Experiment: 5% $\rightarrow$ 8%

We then turn to the baseline experiment that mimics Japanese consumption tax in 2014 in which consumption tax rate was raised from 5% to 8%. At the same time, this section examines how the tax wedge help account for the dynamic pattern of the empirical tax elasticity discussed in the previous subsection 7.1.1. To investigate the roles of frictions, I compare the baseline model with a model without the tax wedge, where  $q = (1 + \tau^c)$ .

In the following experiments, the economy is assumed to be in the stationary equilibrium in the first period, the government announces in the beginning of the second period that the consumption tax is increased in the sixth period, and the households fully understand this announcement.

Figure 8 shows the result of the experiment in which consumption tax is raised from 5% to 8%. All the variables are normalized by the initial stationary equilibrium values (e.g. non-durable consumption in the figure in period  $t$  is  $C_t/C_1$ ). The solid lines depict dynamics of one with the partial irreversibility (baseline) and dashed line show that in the model without the tax wedge, that is when  $q = 1 + \tau^c$ . Regarding the dynamics of the durable expenditure, I report  $X_+^D \equiv \sum_j \int_{x^d > 0} x^d d\mu_j$ .

Figure 9: Other Investment Frictions



Note: Left: Comparison with the Quadratic adjustment cost model. Right: Comparison with Fixed Cost model. The functional form of the quadratic adjustment cost is  $QC(d, d_{-1}) = \frac{QC}{2} [d - (1 - \delta^d) d_{-1}]^2$ , and that of the fixed cost is  $F(d, d_{-1}) = \mathbb{1}_{d \neq 0} F^d (1 - \delta^d) d_{-1}$ .

In the Figure 8, the non-durables stay almost constant in both models. Also, the both models predict a large durable purchase right before the tax hike  $t = 5$  and a large drop in the durable expenditure in common in the period of the tax hike, implying that the stockpiling is due to short-run intertemporal substitution. The two models, however, exhibit distinct predictions in speed of the recovery after the tax hike and the magnitude of stockpiling in durable expenditure  $X_+^D$  in fifth period.

### 7.2.1. Slow Recovery

Looking at the durable expenditure  $X_+^D$  after the tax hike, it can be confirmed the baseline model exhibits gradual recovery. This is because of (S,s) nature resulted from the tax wedge, which works in the way the partial irreversibility constraint functions. That is, once households accumulate enough amounts of durable stocks before the tax hike, they do not adjust the durables for some periods afterwards. This keeps the households to wait inactively until the stockpiled durables depreciate enough. The fact that the partial irreversibility constraint increases the persistence of aggregate expenditure is known in the literature (e.g. Lam (1989)). A new finding here is that higher persistence comes from the slow convergence only after the tax change (or shock) but not before it. That is, persistence is asymmetric before and after the tax change. As discussed, this asymmetric persistence is key observed pattern in empirical tax elasticity of taxable spendings.

Readers may wonder if other investment frictions, for instance fixed-cost and quadratic adjustment cost, works in the same way. This concern is critical because if a model with other frictions behaves in the same manner, the researcher cannot identify the model correctly. To re-

solve the concern, I construct a model that added to the baseline model (i) quadratic adjustment cost,  $QC(d, d_{-1}) = \frac{QC}{2}[d - (1 - \delta^d)d_{-1}]^2$  or (ii) fixed-cost,

$$F(d, d_{-1}) = \begin{cases} F^d(1 - \delta^d)d_{-1} & \text{if } d \neq (1 - \delta^d)d_{-1} \\ 0 & \text{otherwise} \end{cases}$$

which follows [Berger and Vavra \(2015\)](#).  $QC$  and  $F^d$  are fixed parameters that governs degree of adjustment cost respectively.<sup>23</sup>

Figure 9 shows the results from two models. The left panel compares the baseline model and the model embedded with the quadratic adjustment costs. The quadratic adjustment cost model exhibits slow recovery in the aggregate positive durable expenditure  $X_+^D$  but gradual stockpiling in third and forth period which is inconsistent to [Baker et al. \(forthcoming\)](#) for aforementioned reason.

The right panel of Figure 9 shows the results from the fixed-costs model. This model also generates slow recovery but the aggregate durable expenditure  $X_+^D$  dips in third and forth period which is also counter-factual. In addition, the fixed-cost model exhibits overshooting in tenth, fourteenth and eighteenth periods due to negative persistence that fixed-cost generates.

With these properties in the quadratic adjustment and fixed-cost, it seems that the tax wedge (or the partial irreversibility constraint) is the best investment frictions for the study of the consumption tax change.

### 7.2.2. Overprediction

The baseline model predicts lower stockpiling in the  $t = 5$ . Alternatively a model without the tax wedge generates larger stockpiling. It is due to the unrealistic revenue from the tax system in the model without the wedge. If ignoring the tax wedge, the budget constraint is simply

$$(1 + \tau^c)[c + x^d] + a = (1 + r)a_{-1} + y_j(e) + b$$

regardless of sign of  $x^d$ . Therefore, if the household sells durables (i.e.  $x^d < 0$ ), the household receive  $\tau^c x^d$  as a revenue on top of  $x^d$ . This becomes more problematic on the dynamics. To demonstrate the problem, consider a simple example. Suppose  $\tau_{\text{today}}^c = 5\%$  today and  $\tau_{\text{tomorrow}}^c = 8\%$  tomorrow. Importantly, the households in a model without the tax wedge can make an unrealistic revenue by doing the following speculation. That is, if the households buy the durable today and sell it tomorrow, they will make a profit due to tax differential  $\tau_{\text{tomorrow}}^c - \tau_{\text{today}}^c = 3\%$ . This revenue from the tax differential partly offsets the cost of holding the durables i.e., the

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<sup>23</sup> $QC = 0.09$  taken from [Cashin \(2017\)](#) and  $F^d = 0.05$  taken from [Berger and Vavra \(2015\)](#).

depreciation and the cost of large utility fluctuation due to the stockpiling. In this way, the unrealistic revenue induces the speculative stockpiling in the model without the tax wedge.

Notice that this problem is more critical as (i) the tax differential becomes large, (ii) the depreciation rate is low or (iii) the utility cost of fluctuation is low. For instance, if a researcher construct a monthly model, she would set  $\delta^d$  very low. Or, if the researcher introduces very long-lasting goods (e.g., housing) into a model, very low  $\delta^d$  is also set.<sup>24</sup> These discussions mean that choice of frequency of the model and coverage of the durable goods may exacerbate the misspecification problem critically. My model frequency is annual and the model does not include housing so far. That is, the potential problem due to misspecification is more critical and I view my result as a lower bound of the problem.

Cashin and Unayama (2016) and Cashin (2017) estimate intertemporal elasticity of substitution  $1/\sigma$  using a representative household model with durables but without the tax wedge. If true data generation process is my baseline model, their models overpredict the stockpiling behavior due to the misspecification. This overprediction must lead to a bias in their estimation results. For instance, Cashin and Unayama (2016) should have the lower IES than true value due to the misspecification.

### 7.2.3. Stockpiling with Different Magnitude of the Tax Hike

The previous subsection discussed the speculative stockpiling largely depends on the degree of the tax hike. This subsection quantitatively study how much the stockpiling behavior can be biased due to the misspecification problem with different scale of the tax hike.

Before demonstrate it, let be clear about how this section examines the speculation. As discussed in previous sections, the stockpiling in the benchmark model takes place (i) in the form of durables only (ii) only one period before the tax hike. Thus, the Figure 10 reports  $X_+^D(5)/X_+^D(1)$  for different new tax rates  $\tau_{\text{new}}^c = 6, 7, \dots, 15\%$ . The Figure 10 reports results of both baseline model and the model without the tax wedge.

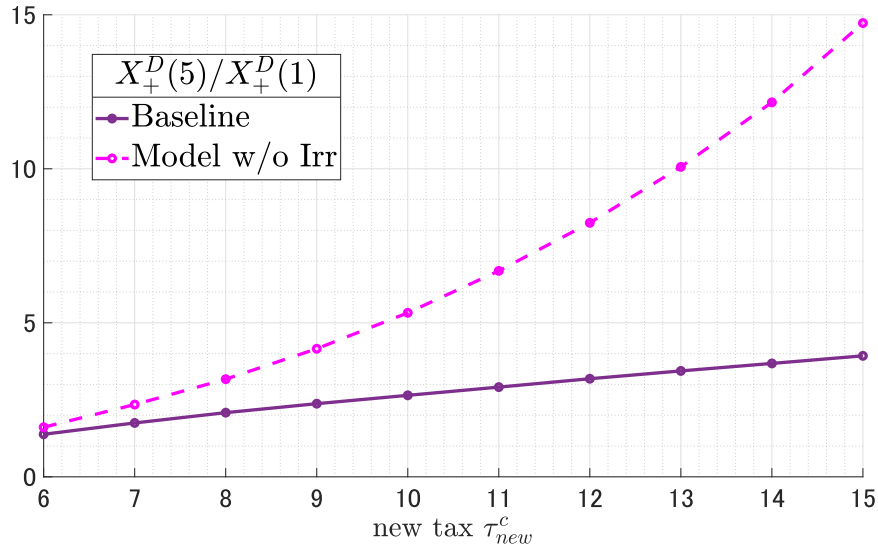
In the Figure 10, two models generate clearly distinct predictions. On the one hand, the baseline model predicts relatively moderate and almost linearly increasing stockpiling in the new tax rate. On the other hand, the model without the tax wedge shows far large and non-linearly increasing stockpiling. Note that since the scale is  $X_+^D(5)/X_+^D(1)$ , 2 in the y-axis, for instance, means double but not 2 % increase. This large difference cannot be ignored for the analysis of tax hike.

This analysis suggests an application of this work to future fiscal problem in Japan. Government of Japan is advised by IMF to increase consumption tax rate to 20% in the end (See

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<sup>24</sup>The purchase of houses is subject to consumption tax in Japan.

Figure 10: Stockpiling over  $\tau_{\text{next}}^c = 6, 7, \dots, 15\%$ .



Note: The tax in period 1 is  $\tau_{\text{old}}^c = 5\%$ . Then, the tax is increased to  $\tau_{\text{new}}^c = 6, 7, \dots, 15\%$  in  $t = 6$ . The lines depict how much stockpiling of durable  $X_+^D(5)/X_+^D(1)$  occurs

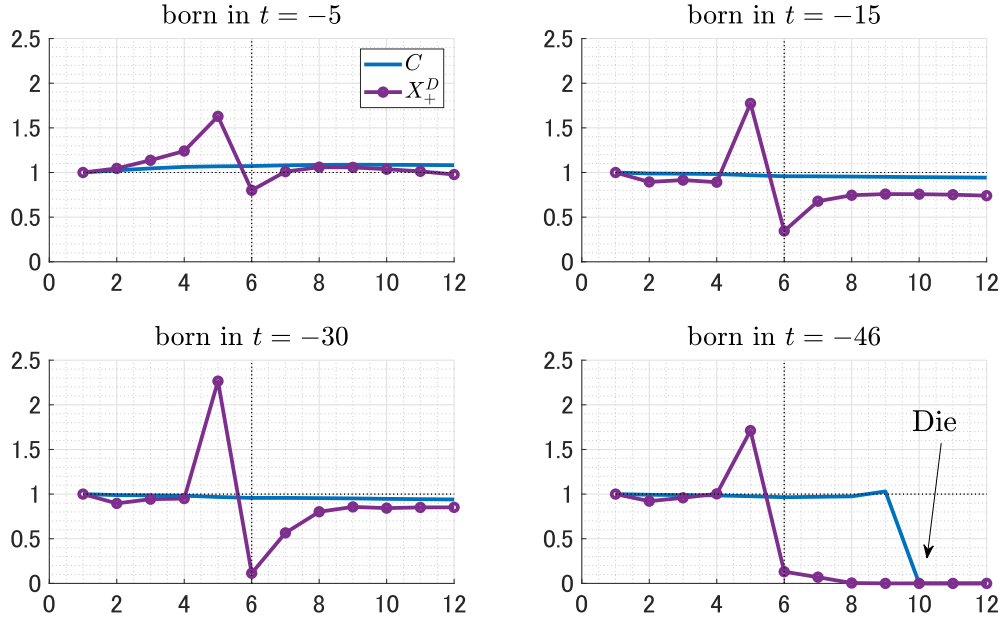
Georgieva (2019)). Also, macroeconomic research (e.g. Braun and Joines (2015) and Hansen and İmrohoroglu (2016)) suggest the need of further consumption tax increase to around 30%. Thus, it is not unrealistic that Government of Japan need to implement large scale consumption tax hike at some point. To the credibly assess the large scale consumption tax hike, I argue the model with the tax wedge is necessary.

#### 7.2.4. Age-Dependency

There are, in theory, several age-dependent effects in this dynamics. First, as shown in Figure 3, different generations have different amount of assets. Cash-on-Hands are essential for the stockpiling and thus the life-cycle asset profile determines the degree of stockpiling for each generation. Second, different generations have different degree of needs of the durables. For instance, most of the young need the durables immediately because they do not own them and thus have the high marginal utility of the durables. The middle-age and old have weaker motive in this respect. Third, different generations pay the higher consumption tax rate for different duration of periods. That is, the young live longer under the new tax rate than the old and thus they pay the the high tax rate longer. This, in turn, implies the young incurs the heavier burden of the tax hike.

Figure 11 shows four generations consumption spending responses. A top-left panel in Figure 11 illustrates the response of those who are born in  $t = -5$  (i.e. age of  $11 + 25 = 36$  in  $t = 5$ ).

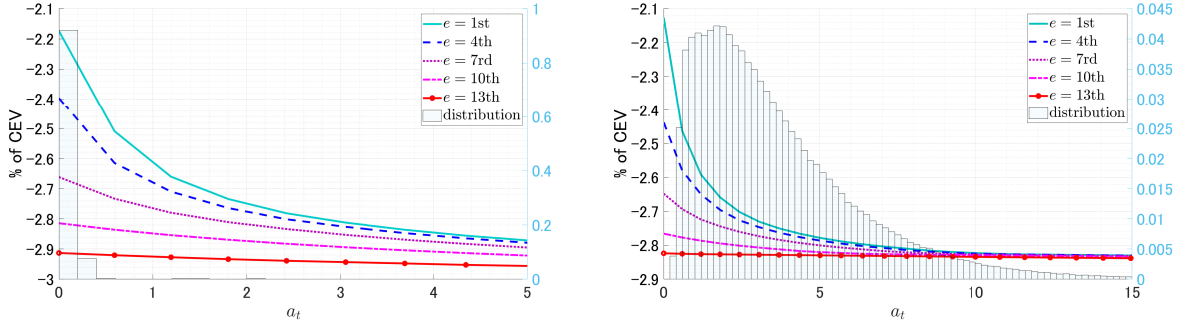
Figure 11: Age-Dependent Responses to the Consumption Tax Hike 5%  $\rightarrow$  8%



This generation of households show relatively weak response to the anticipated consumption tax hike. There are two reasons for this. First, the young have high marginal utility of the durables soon and purchased large amount of durables not only in  $t = 5$  but also  $t = 1, 2, 3, 4$ . That is, the denominator of the variable shown  $X_+^D(t)/X_+^D(1)$  is large. Second, this generation of the households do not own liquid assets  $a$  much, which prevent them from stockpiling large amount of the durables. As a result, this generation of households do not show the strong stockpiling in the panel. Next, a top-right panel is expenditure responses of those who are born  $t = -15$  (i.e. who are age of  $21 + 25 = 46$  at  $t = 5$ ). This generations of households illustrate strong stockpiling behavior for two reasons. First, since the average of this generation of households are asset-wealthy, they afford the durable more freely than the young. Second, this households already accumulated enough durable stocks and thus did not have strong needs for the durables in period  $t = 1$  unlike the young generation. Therefore, this households stock up the large amount of durables for the short-term intertemporal substitution motive in  $t = 5$ . A bottom-left figure shows another middle-age generation who are born in  $t = -30$  (i.e. who are age of  $36 + 25 = 61$  at  $t = 5$ ). The responses of this middle-age generation in  $t = 5$  is stronger than those of who are born in  $t = -15$  (top right). This is because they are very rich in asset and thus buy the luxurious durables more than those who are born in  $t = -15$ . Finally, a bottom-right figure demonstrates how the old (who are age of  $51 + 25 = 76$  in  $t = 5$ ) react to the tax hike. Because of the high



Figure 12: Welfare Changes by Consumption Equivalent Variation



Note: Each line depicts welfare costs of the tax hike 5%  $\rightarrow$  8%. The histogram behind the lines shows the asset distribution in the stationary equilibrium. The households with higher productivity incur higher welfare costs of the tax hike. Also, the asset wealthy households lose more.

durable depreciation rate, this old households still stockpile the durables to a certain degree.

Overall, Figure 11 confirms that stockpiling behavior is highly age-dependent and most of the aggregate stockpiling behavior is explained by the middle-age generations.

#### 7.2.5. Welfare Analysis

Next, I analyze welfare changes over the transition with consumption equivalent variation (CEV, hereinafter). The welfare evaluation over the transition is of the first order interest in this paper. The tax-induced inflation effect, for instance, only appears over the dynamics.

Figure 12 shows the welfare changes of two generations, age of 30 at  $t = 1$  and age of 45 at  $t = 1$ , along asset dimension and idiosyncratic shock over the transition. Overall, all the households in the Figure 12 incur the welfare costs, simply because the policy change we consider here is tax hike with no transfer. What this paper is interested in here is which households loose more.

First, lines in both left and right panels of the Figure 12 show that asset poor households lose less. This is because the tax-induced inflation effect. Asset wealthy households see large decline in their assets value, and thus reduces the consumption more than the asset poor. In other words, the consumption tax hike is a policy tool to tax preexisting wealth. Also, both of the panels confirms that highly productive households lose more as well. This is negative income effect. That is, given the high persistence of labor productivity, the highly productive households lose the future potent high wage. This results in the larger decline in the consumption of high earners. These analysis does not support regressivity of the consumption tax burden, which many of popular discussions argue.

Second, I find that young households potentially lose more by comparing two panels in 12.

That is, the young households in the left panel see more than 2.7% decline in the CEV while the middle-age households in the right panel see less than 2.7%. This is because the young live longer in the remaining life and thus see a larger decline in the present value of labor earnings.

Third, these lines only shows the decline in the welfare given state spaces  $(a, d, e, j)$ , but does not reflect the distribution. Thus, I overlay the distribution on the both of panels. The distributions clearly show that there is no asset wealthy young households in the model thus only a few young households loose  $-2.8\%$  in CEV while many of middle-age households accumulate the asset and thus face  $-2.8\%$  decline in CEV.

Overall, welfare analysis shows burden of the consumption tax is highly heterogeneous, and asset wealthy and highly productive young households incur the largest burden of the consumption tax hike.

### 7.3. Consumption Tax Decrease: Asymmetry

This section examines a counter-factual experiment in which the tax is decreased. The result of this analysis can be insightful for the recent policies implemented in Germany and U.K in 2020 in order to combat the on-going pandemic crisis. As the model features the high non-linearity, this section confirms that the tax decrease is highly asymmetric to the tax hike.

Figure 13 shows the results of when consumption tax is decreased from 8% to 5%. For the comparison, I also depict the result when tax is increased from 5% to 8%. The tax decreases is highly asymmetric to the tax hike. As the households know the future tax decreases, they gradually decreases  $X_+^D$  to postpone the purchase by letting the durable stock depreciate. Then, they refill the stock of durables once the tax is decreased in period  $t = 6$ .

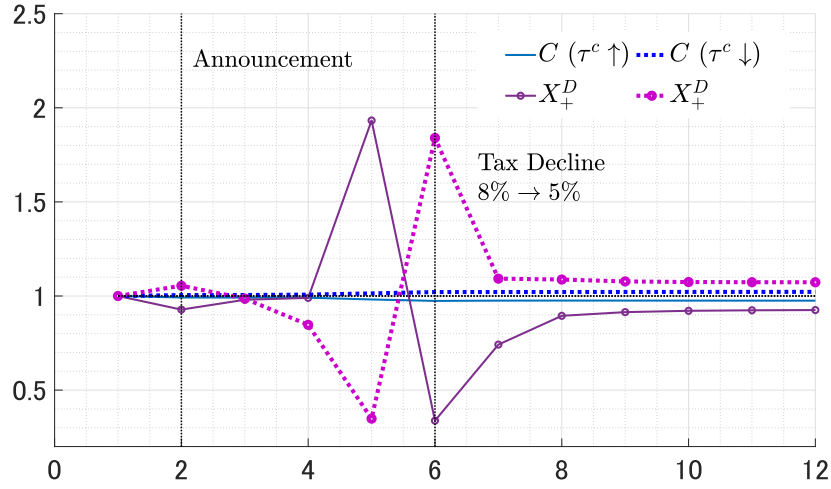
This asymmetry suggests one lesson for empirical strategy. Suppose an econometrician runs a regression  $\Delta \log(y_t) = \sum_i \beta_i \Delta \log(1 + \tau_{t+i}^c) + z_t + \epsilon_t$ , where  $y_t$  are taxable spendings or durable purchases,  $z_t$  here is a sum of controls and fixed effects, and  $\beta_i$  is a coefficient of interest.<sup>25</sup> If the econometrician separate sample into the tax hike and decrease cases, the resulting coefficient  $\beta$  in the lead lag is hard to interpret. Thus, the theory suggests that the tax hike and decrease be separately analyzed.

Another implication of asymmetry is that linearized solution is not appropriate for the study of consumption tax changes. If the model is linearized, then the solution does not capture the asymmetry.

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<sup>25</sup>For instance, see Baker et al. (2019), and Baker et al. (forthcoming) for the regression analysis in this literature.

Figure 13: The Tax Hike vs Decrease 8%  $\rightarrow$  5%.



Note: The dashed lines illustrate the household expenditures when the tax is decreased. This counter-factual experiment shows that the tax decrease has the asymmetric effects compared with the tax hike.

#### 7.4. One-Time vs Two-Times

Many of the economic downturn are persistent. When considering the tax hike as the aggregate demand stimulus policy in the recessions as [Feldstein \(2002\)](#) and [Hall \(2011\)](#) proposed, the natural concern is whether the policy simply induces the short-run intertemporal shifting of the households expenditures. If the consumption tax hike induces the short-run intertemporal demand shifting only, the economy may see the plummet of the households expenditure in the midst of the recovery from the crises. Thus, a natural question is how to design the policy so that it causes longer intertemporal shifting.<sup>26</sup>

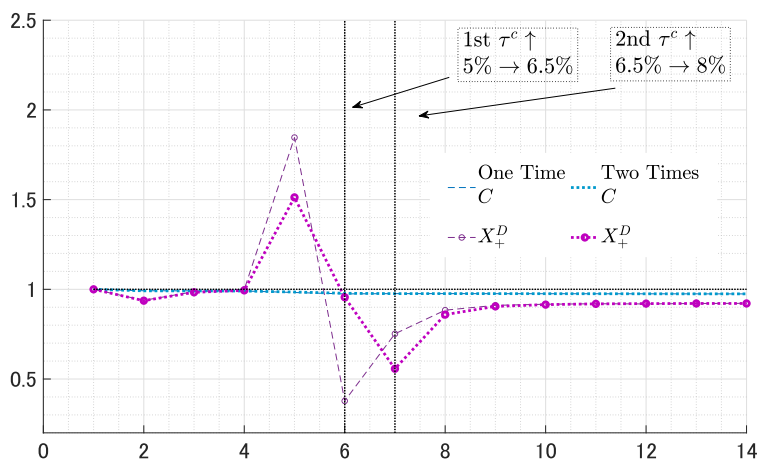
To answer the question, this section compares two experiments: an experiment with one time tax hike and the other with two tax hikes. The idea borrows from the original proponent of the unconventional fiscal policy, [Feldstein \(2002\)](#). If the government keeps increasing the tax successively, the households do not stop purchasing the durable right after the first tax hike.

Figure 14 simulates the both experiments. In the one time hike case, the government increases the tax to 8% in  $t = 6$ . The effects of the tax hike is similar to previous section. When the government increases two times, I assume it first increases to 6.5% in  $t = 6$ , and then 6.5% to 8% in  $t = 7$ .

Figure 14 illustrates the effects of two tax hikes. The stockpiling behavior in the two times

<sup>26</sup>Also, Government of Japan increased consumption tax rate with two steps: from 5% to 8% in 2014 and 8% to 10% in 2019. This Japanese experience is one motivation for this experiment.

Figure 14: One Time Hike vs Two Times Hikes.



Note: In one time case: the government raises the tax rate 5% to 8% in  $t = 6$ . In two times case: the government raises it 5%  $\rightarrow$  6.5% in  $t = 6$  and 6.5%  $\rightarrow$  8% in  $t = 7$ .

hikes case is a bit weaker than one time. However, importantly, the drop in the durable expenditure after the tax hike gets (i) moderate and (ii) is postponed to the future. This result imply that if the government keep increasing the tax hike until the end of the recession, it can postpone the timing of the drop in the households expenditure.

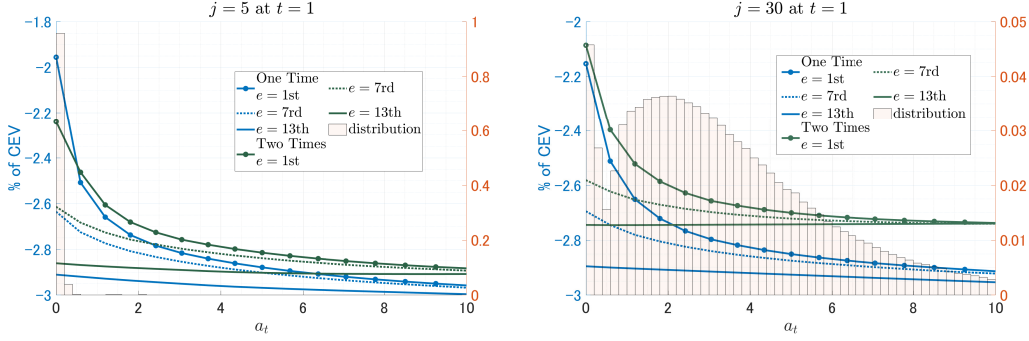
Also, welfare analysis in Figure 15 indicates the two-times tax hike scheme is welfare improving for almost all of the households except young borrowing constrained households. The welfare gains comes from the fact that the households change the durable stock less in the two-times tax hike case. Thus, they do not incur the utility costs of violation of the consumption smoothing due to the large stockpiling behavior.

## 8. Short-Run: General Equilibrium

My baseline tax hike experiment is to raise the tax by 3% and this experiment is arguably a macroeconomic-scale policy change. Therefore, it seems natural to consider how macroeconomic variables including prices respond to the policy change. This section finally shows the results from the general equilibrium and compare them with the partial equilibrium results. It turns out that the equilibrium assumption does not alter the results much. This section discuss why two equilibrium assumptions predict the very similar consequence.

Figure 16 compares the dynamics of partial and general equilibrium. Overall, the dynamics

Figure 15: Welfare Costs of One Time Hike vs Two Times Hikes.



Note: In one time case: the government raises the tax rate 5% to 8% in  $t = 6$ . In two times case: the government raises it 5%  $\rightarrow$  6.5% in  $t = 6$  and 6.5%  $\rightarrow$  8% in  $t = 7$ .

of the two look very similar. The main reason why general equilibrium shows the similar results of the partial equilibrium is low interest rate elasticity of the aggregate durable expenditure. Koby and Wolf (2020) and Winberry (2020) analytically shows that interest rate elasticity is the sufficient statistics for whether the results of the general equilibrium is close to those of the partial equilibrium. The intuitive explanation follows that the general equilibrium is the environment where prices move. Therefore, if the aggregate quantities respond to the price movement only a little due to the low elasticity, the general equilibrium effects are tiny.

Recent literature argue that standard (S,s) models generate infinitely high interest rate elasticity. (e.g., Koby and Wolf (2020), McKay and Wieland (2019) and Winberry (2020)) In my benchmark model, the model-generated interest rate elasticity is as low as 4.1. On the contrary, many of fixed-cost models generate the elasticity of 50 – 1,075.<sup>27</sup> Empirical literature on the durables shows the interest elasticity is between 1.1 – 5.0 (Baker et al. (2019), Mian and Sufi (2012)). Thus, the mechanism of lowering the model-implied interest rate elasticity is of interest in the lumpiness literature. The baseline model generates the low elasticity for the following three reasons.

Successfully matching the interest rate elasticity is crucial not only for the equilibrium concept but also for the tax change experiment because the consumption tax change has a profound connection with the change in the interest rate as the Euler equation below shows,<sup>28</sup>

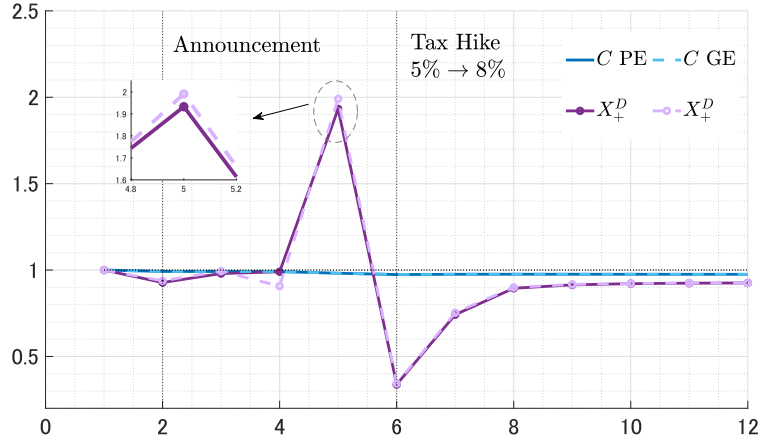
$$u'(c) \geq \beta(1 + r') \frac{1 + \tau^c}{1 + (\tau^c)'} u'(c').$$

Thus, the tax change has a very similar implication to the interest rate in my baseline model as

<sup>27</sup>See Koby and Wolf (2020) and Winberry (2020) for the lumpy investment, and McKay and Wieland (2019) for the lumpy durables.

<sup>28</sup>I omit the durables from the Euler equation here since the Stone-Geary is the additive separable when it is monotonically transformed.

Figure 16: Partial Equilibrium and General Equilibrium



*Note:* The dashed line shows the household expenditure in the general equilibrium where  $(r, w)$  clear the markets. The general equilibrium does not alter the quantitative results significantly.

well. As a result, replicating the low elasticity in line with the empirical studies is key for the main focus of this study.

First, and most importantly, it turns out that life-cycle motives dampens the interest rate elasticity. This is first because there is a mass of households who adjusts durables regardless of the change in interest rate, which reduces the interest sensitivity of the durable expenditure in a model. For instance, the young buy the durables because of the infinitely high marginal utility while the old do not buy the durable because they will die soon. In this way, the model implied interest elasticity of the aggregate durable expenditure is dramatically lowered.

Second, two asset structure is key to reduce the elasticity, as explored in [Berger and Vavra \(2015\)](#). That is, if the agents have only one asset as in [Khan and Thomas \(2008\)](#), the good market clearing condition  $Y = C + I_k$  implies that a change in  $I_k$  propagates a change in  $C$ . At the same time,  $C$  is governed by the household smoothing motive in the equilibrium. Thus strong households consumption smoothing motive smooths the investment  $I_k$  through the good market clearing condition or large price adjustment. By contrast, if the households have two assets, a lumpiness in one type of assets may not be smoothed by the consumption smoothing motive because households now have an additional measure to smooth consumption. In this way, the two asset structure weakens the elasticity.

Third, since I do not include housing in my analysis, the depreciation of the durables is higher than the value typically used in the literature. As [House \(2014\)](#) discusses, the higher the depreciation is, the lower the interest sensitivity is. This is because, if the depreciation is

high, the households make durable spendings for a short-term use. This, in turn makes the durable expenditure frequent and small-scale. As a result, the high depreciation rate reduces the sensitivity of the durable expenditure to the change in the interest rate.

## 9. Concluding Remarks

This paper is, to my best knowledge, a first paper that studies the stockpiling response to the preannounced consumption tax changes using a life-cycle heterogeneous-household general equilibrium model of durables and the tax wedge. The tax wedge is introduced to reflect the actual consumption tax system that households face only when buying the durables but not when selling them. Using the novel model, this paper explores the effects of anticipated consumption tax changes.

The model replicates the dynamic pattern of the tax elasticity of taxable spending very well, owing to presence of the tax wedge. I also show that either a frictionless model or models with other types of investment frictions (i.e. convex adjustment costs, and fixed-costs) do not reproduce the dynamic pattern. Second, life-cycle structure is key to lower the elasticities. Combined with the tax wedge and life-cycle structure, my baseline model accounts for the empirical tax elasticities of taxable spendings and durable expenditures over the dynamics well.

Also, the tax wedge is qualitatively necessary to generate stockpiling behavior with true stockpiling motive in response to a preannounced tax change. Without the tax wedge, the resulting stockpiling is not only due to households own use but also for the speculation. Thus, the model without the tax wedge largely overpredicts the stockpiling behavior. If researcher estimates structural parameters (e.g., elasticity of intertemporal substitution) using consumption tax hike and the model without the tax wedge, the estimated parameters must be biased due to the speculative stockpiling.

Welfare analysis finds that the tax hike over the transition is progressive. This is because the increase in the consumption tax rate imposes a tax on the preexisting wealth. In my baseline model, the presence of the durables further widens the welfare costs inequality.

Finally, I conduct two counter-factual experiments. The first experiment is the consumption tax decrease. I find that consumption tax decrease is asymmetric to the tax hike. In the tax decrease case, many of the households stop buying the durables far before the tax hike and let the durable depreciate until the period of tax decrease. The second experiment is the multiple-times tax hike. The multiple-times tax hike induces smoother intertemporal shifting in the household expenditure and thus welfare improving for most of the households except young borrowing



constrained households. Overall, the paper explored effects of the anticipated consumption tax with the novel model.

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## 10. Appendix A: Computation

### 10.1. Nested EGM Algorithm

This section explains algorithm of the Nested EGM used in the paper. The algorithm is originally developed by [Drue Dahl \(2020\)](#) and I extend it so as to incorporate the partial irreversibility constraint into the model. The critical advantage of this algorithm is the *nesting* assumption which reduces the dimensions of the maximization problem in choice-specific value functions. I start with the exposition of the nesting assumption. For simplicity, I focus on the solution of stationary equilibrium where prices  $(r, w)$  and the tax  $\tau^c$  are fixed. Extending the algorithm to the dynamics is straightforward.

Also, the algorithm is general enough to have fixed-cost without any additional computational efforts. Thus, I add the following form of fixed-cost here:

$$F(d, d_{-1}) = \begin{cases} F^d(1 - \delta^d)d_{-1} & \text{if } d \neq (1 - \delta^d)d_{-1} \\ 0 & \text{otherwise} \end{cases}$$

where  $F^d$  is a parameter. This is a common functional form in the lumpy durable literature (e.g., Berger and Vavra (2015), McKay and Wieland (2019)).

In order to apply the Nested EGM, we need to rewrite the model in the following way. The notation here from now on follows the old version of Druedahl (2020).<sup>29</sup> Let  $m$  be a cash-on-hand, and which is defined as

$$m = (1 + r)a_{-1} + y_j,$$

$$\text{where } y_j = (1 - \mathbb{1}_{j < J^R})w\kappa_j e + \mathbb{1}_{j < J^{RSS}} + b.$$

where  $a_{-1}$  is the asset stocks at the end of previous period and  $y_j$  is the either labor earning or the social security depending on the age plus received bequest.

In the beginning of every period, households own a set of  $(m, \bar{d}, e)$  and compute

$$m^{up} = m + (1 + \tau)(1 - \delta^d)\bar{d} - F^d(1 - \delta^d)\bar{d}$$

$$m^{down} = m + q(1 - \delta^d)\bar{d} - F^d(1 - \delta^d)\bar{d}$$

where  $m^{up}$  and  $m^{down}$  are *choice-specific cash-on-hands* for *up* and *down* respectively, and  $\bar{d}$  is a stock of the durable at the beginning of the period.

Given these cash-on-hands  $(m, m^{up}, m^{down})$ , the households compute choice-specific value functions  $(v^{inact}(\cdot), v^{up}(\cdot), v^{down}(\cdot))$  in the midst of the period. Particularly, I assume that the inaction problem is solved first. This is the assumption *nesting* the timing within a period so that the households first decide how much non-durable consume, and then decide later how much durable.

$$v_j^{inact}(m, \bar{d}, e) = \max_{a \geq 0} u(c, (1 - \delta^d)\bar{d}) + w_j(a, (1 - \delta^d)\bar{d}, e)$$

$$\text{s.t. } (1 + \tau^c)c + a = m.$$

Given the choice of the non-durable consumption  $c^{inact}(\cdot)$ , the households choose intensive margin of the durable. When upward adjusting, the households use this consumption function

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<sup>29</sup>The title of his old version was "A Fast Nested Endogenous Grid Method for Solving General Consumption-Saving Models."

and solve the problem below,

$$\begin{aligned}
v_j^{up}(m, \bar{d}, e) &= \max_d u(c, d) + w_j(a, d, e) \\
\text{s.t. } (1 + \tau^c)[c + d] + a &= m^{up} \\
c &= c^{inact}(m^{up} - (1 + \tau)d, d, e) \\
d &> (1 - \delta)\bar{d}.
\end{aligned}$$

Notice that the dimension of the maximization is only one, thanks to the nesting assumption. This significantly reduces the computational burden and we are not bothered by the curse of the dimensionality to solve this optimization problem.<sup>30</sup>

When downward adjusting,

$$\begin{aligned}
v_j^{down}(m, \bar{d}, e) &= \max_d u(c, d) + w_j(a, d, e) \\
\text{s.t. } (1 + \tau^c)[c + d] + a &= m^{down} \\
c &= c^{inat}(m^{down} - qd, d, e) \\
d &< (1 - \delta)\bar{d}
\end{aligned}$$

Also notice that these Bellman equation does not contain  $v_{j+1}(\cdot)$ . We used instead *post-decision value function*  $w_t(\cdot)$  and its domain is different from value function  $v_j(\cdot)$ . The inclusion of post-decision value function allows pre-computation.

This post-decision value function is defined as

$$\begin{aligned}
w_j(a, d, e) &= \beta \mathbb{E}[v_{j+1}(m', \bar{d}', e') | e] \\
m' &= (1 + r)a + y_{j+1}.
\end{aligned}$$

We are ready to explain the algorithm.

- (i) **Grid spacing:** Space the grids for  $a, \bar{d}, m, m^{up}, m^{down}$ . In practice, we use the same grid for  $m^{up}$  and  $m^{down}$  but they are conceptually different.<sup>31</sup>

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<sup>30</sup>In practice,  $w_j(\cdot)$  can be a very kinky function due to the discrete choices. Thus, I use many multi-starts and a local-maximization method for this problem.

<sup>31</sup>It is not computationally costly to use a large number of grid points for  $(a, m)$  because the inaction problem is solved fast. But an increase in the number of grid points for  $\bar{d}$  is very costly.

(ii) **Last period problem:** Solve the last period problem.<sup>32</sup> That is, for any  $(m, \bar{d}, e)$ ,

$$(1 + \tau^c)c = m$$

$$v_J(m, \bar{d}) = u\left(\frac{m}{1 + \tau^c}, \bar{d}\right).$$

(iii) **Loop for age:** For any  $j = J - 1, \dots, 1$ , repeat the below.

(i) **Inaction Problem:**

(i) **Pre-computation:**

(i) **Next Period Values:** For any  $(e, a, \bar{d}')$ , compute<sup>33</sup>

$$m' = (1 + r)a + y_{j+1}$$

$$(m^{up})' = m' + (1 + \tau^c)(1 - \delta^d)\bar{d}' - F^d(1 - \delta^d)\bar{d}'$$

$$(m^{down})' = m' + q(1 - \delta^d)\bar{d}' - F^d(1 - \delta^d)\bar{d}'$$

(ii) **Interpolations:** Evaluate the value functions on  $(e, a, d)$ . That is, compute<sup>34</sup>

$$v_{j+1}^{inact}(e', a, \bar{d}') = v_{j+1}^{inact}(e', m', \bar{d}'')$$

$$\text{where } \bar{d}'' = (1 - \delta^d)\bar{d}'$$

$$v_{j+1}^{up}(e', a, \bar{d}') = v_{j+1}^{up}(e', (m^{up})', \bar{d}')$$

$$v_{j+1}^{down}(e, a, \bar{d}') = v_{j+1}^{down}(e', (m^{down})', \bar{d}')$$

and then take a maximum of the discrete choice

$$v_{j+1}(e', a, \bar{d}') = \max\{v_{j+1}^{inact}(\cdot), v_{j+1}^{up}(\cdot), v_{j+1}^{down}(\cdot)\}.$$

and here store the unconditional *next-period* consumption function  $c'(e, a, \bar{d}')$  and durable function  $d'(e, a, \bar{d}')$ .

(iii) **Post-Decision Value Function:** For  $(e, a, d)$ ,

$$w_j(e, a, \bar{d}') = \beta \mathbb{E}[v_{j+1}(e', a, \bar{d}')|e]$$

$$q_j(e, a, \bar{d}') = \beta(1 + r)\mathbb{E}_j[\alpha(c')^{\alpha(1-\rho)-1}(d')^{(1-\alpha)(1-\rho)}].$$

<sup>32</sup>If solving an infinitely lived agent model, this computation can be also applicable if you set  $v^0 = 0$ .

<sup>33</sup>Notice that  $\bar{d}'$  is the durable stock at the beginning of next period, and  $d$  is the durable stock at the end of this period. Thus,  $d = \bar{d}'$ . Also,  $a$  is asset stocks at the end of the period.

<sup>34</sup>Regarding  $\bar{d}''$ , we need to count the depreciation here only for inaction because depreciation for *up* and *down* are already taken into account in  $m^{up}$  and  $m^{down}$ .



- (ii) **EGM:** Apply EGM to the inaction problem. Here, notice  $d = \bar{d}'$  is used due to the inaction.

$$c(e, a, \bar{d}') = u^{-1} \left( \frac{q(e, a, \bar{d}')}{\alpha(\bar{d}')^{(1-\alpha)(1-\rho)}} \right)^{\frac{1}{\alpha(1-\rho)-1}}$$

$$m(e, a, \bar{d}') = (1 + \tau^c)c(e, a, \bar{d}') + a$$

The  $m(e, a, \bar{d}')$  is the *endogenous grid* for the cash-on-hand.

- (iii) **Borrowing Constraint:** This part is standard. Denoting  $m$  as an exogenous grid point, if  $m(e, a, \bar{d}') \geq m$ , then set

$$(1 + \tau^c)c = m$$

$$v^{inact}(e, m, \bar{d}') = u(c, \bar{d}') + w(e, 0, \bar{d}')$$

- (iv) **Upper Envelop Algorithm (and Interpolations):** Let  $ia$  be the  $ia$ -th grid point of grid  $a$ . Similarly,  $ie, im, id$  are defined as well. We can create the following function,

$$c^*(ie, ia, id, im) = c(ie, ia, id) + \frac{m(im) - m(ie, ia, id)}{m(ie, ia + 1, id) - m(ie, ia + 1, id)}(c(ie, ia + 1, id) - c(ie, ia, id)),$$

where  $m(im)$  is  $im$ -th exogenous grid point value. Subsequently define  $a^* = m - (1 + \tau^c)c^*(ie, ia, id, im)$ , and then

$$w^*(ie, ia, id, im) = w(ie, ia, id) + \frac{a^* - a(ia)}{a(ia + 1) - a(ia)}(w(ie, ia + 1, id) - w(ie, ia, id)).$$

Finally, take the max over  $a$  as follows

$$v_j^{inact}(ie, im, id) = \max_{ia} u(c^*(ie, ia, id, im), d(id)) + w^*(ie, ia, id, im).$$

The obtained choice-specific value function  $v_j^{inact}(\cdot)$  here is the globally maximizing choice-specific value function. Also, the consumption function for the inaction problem  $c_j^{inact}(\cdot)$  is found here and will be used to solve up and down problems. Note that this procedure does not take so much time, because these maximization are taken only over the candidates that satisfies necessary conditions for maximization (i.e. the Euler equation with inequality and the budget constraint), but not over entire  $a$ .

(ii) **Upward Adjusting Problem:** Solve the upward adjusting problem with standard VFI.

$$\begin{aligned}
v_j^{up}(e, m^{up}, \bar{d}) &= \max_d u(c, d) + w_j(e, a, d) \\
\text{s.t. } (1 + \tau^c)[c + d] + a &= m^{up} \\
c &= c^{inact}(e, m^{up} - (1 + \tau)d, d) \\
d &> (1 - \delta)\bar{d}
\end{aligned}$$

From practical perspective, kinks in the value function are caused by the discrete choice and amplified through  $v_{j+1}$  over the iteration of age. Thus,  $w_j$  is supposed to be non-differentiable and have many local-maxima. To deal with this, we separate the grid  $d$  very finely, compute local maximum in the subinterval, and take the maximum of the local maxima.

(iii) **Downward Adjusting Problem:** Similarly, solve the downward adjusting problem with standard VFI.

$$\begin{aligned}
v_j^{down}(e, m^{down}, \bar{d}) &= \max_d u(c, d) + w_j(e, a, d) \\
\text{s.t. } (1 + \tau^c)c + qd + a &= m^{down} \\
c &= c^{inact}(e, m^{down} - qd, d) \\
d &< (1 - \delta)\bar{d}
\end{aligned}$$

Almost all of the codes are written in C++ and they are called by MATLAB through MEX compilation. When solving upward and downward adjusting problem with value function iteration, I used NLopt Library for the optimization toolbox. I tried several optimization methods and find that the Subplex algorithm is the best for solving this model in practice.

## 10.2. Perfect Foresight Path of The Life-cycle Model

Compared with solving the perfect foresight equilibrium path of an infinitely-lived agent model, it is by far more computationally costly to solve that of the life-cycle model. This is because there exist  $J$  generations of households at any periods  $t \geq 0$ .

This implies that all the generations Bellman equations must be solved to solve the partial equilibrium dynamics. Let be  $C_{j,t} \equiv \int c_{j,t}(a_{-1}, d_{-1}, e) d\mu_j$  and as follows. Then the quantities for

all the ages  $(C_{j,t}, D_{j,t}, A_{j,t})_j$  are aggregated at every period, i.e.,

$$C_t = \sum_{j=1}^J C_{j,t}, \quad D_t = \sum_{j=1}^J D_{j,t}, \quad A_t = \sum_{j=1}^J A_{j,t}.$$

Notice that there are some generations who are born after the tax change and some who die before the tax change. Thus, I must solve the  $T + J - 1$  Bellman equations conditional on  $(w, r)$  where  $T$  is the time horizon for the simulation.

My solution method to solve the perfect foresight follows [Nishiyama and Smetters \(2005\)](#) and [Nishiyama and Smetters \(2014\)](#), except that I use stochastic simulation for solving the distribution.

- (i) Choose sufficiently long simulation periods  $T$  so that the economy reach the new steady state at  $T$ . Guess a path of prices  $\{r_t^0, w_t^0\}_{t=1}^T$ .
- (ii) For all the generations  $i = 1, \dots, J + T - 1$ , solve the Bellman equation from  $J$  to 1. Then, get  $\{c_{i,t}, d_{i,t}, a_{i,t}\}$ .
- (iii) Aggregate the individual quantities for those who live at time  $t$

$$C_t = \sum_{j=1}^J C_{j,t}, \quad D_t = \sum_{j=1}^J D_{j,t}, \quad A_t = \sum_{j=1}^J A_{j,t}.$$

- (iv) Check if the markets clear for all  $t$ , that is

$$A_t = K_t.$$

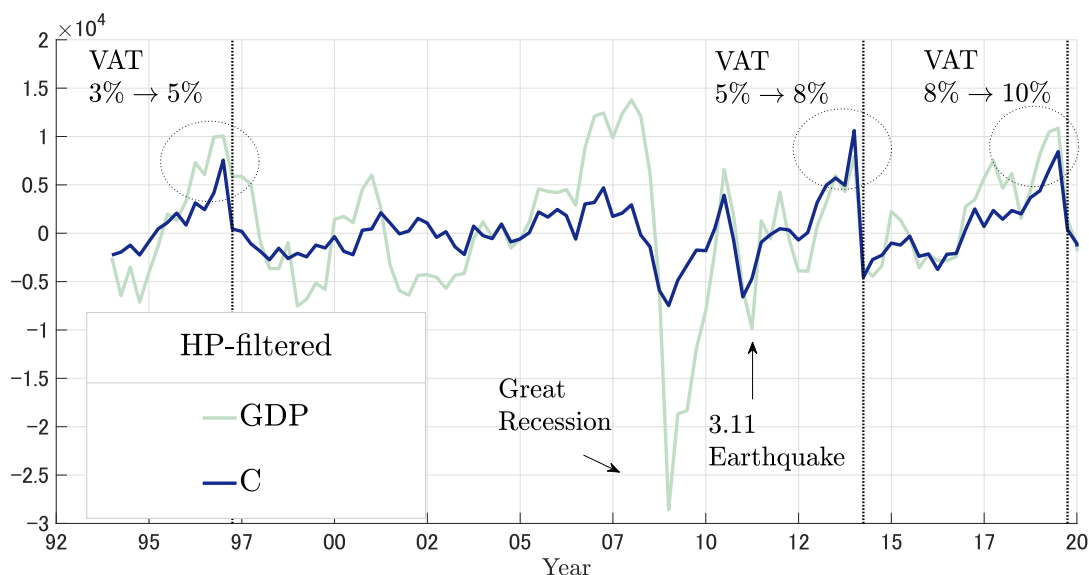
Note that since the labor supply is inelastic, the labor market is cleared by construction. By Walras's law, the good market clears if the asset market clearing condition is satisfied. If the markets are not cleared, update prices  $\{r_t^{\ell+1}, w_t^{\ell+1}\}_{t=1}^{\infty}$  and iterate the procedure until the markets clear.

## 11. Appendix B: Japanese Consumption Tax Hike

This paper applies the baseline mode to the Japanese economy. This section discusses the basic effects of the consumption tax changes in Japan.

Figure 17 shows the cyclical components of HP filtered GDP and aggregate consumptions since 1994 Q1. Aggregate consumptions have been smoother than GDP for most of the time in

Figure 17: Cyclical Components of GDP and Aggregate Consumption in Japan



Note: HP filtered GDP and aggregate consumptions since 1994 Q1 until 2020 Q1. The aggregate consumptions have been smooth except some periods, such as consumption tax changes or large events (e.g. Great Recession and The Tohoku Earthquake). Data source: the national account.

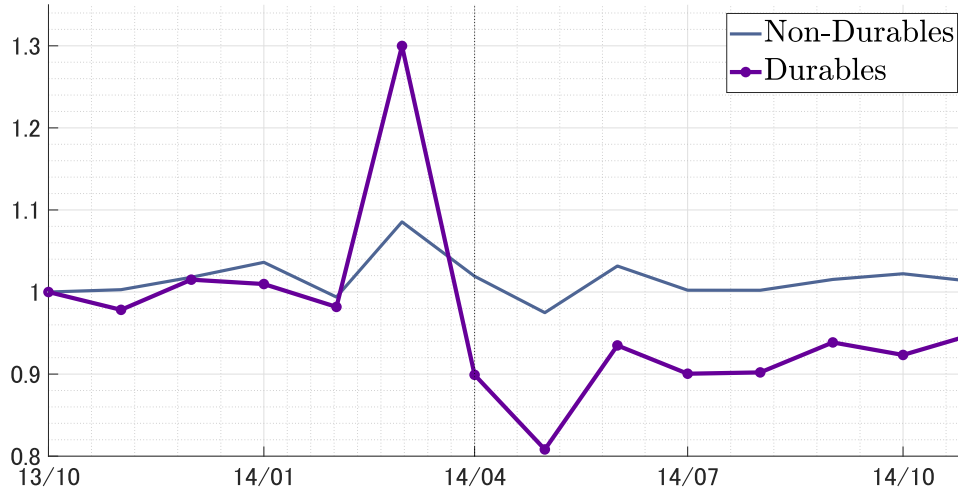
the sample periods, but they sometimes largely swing. For example, the consumptions plunged in 2008 and 2011 because of the Great Recession and the Tohoku earthquake. Also, it can be noticed that large spikes in aggregate consumptions are accompanied with the consumption tax changes in Japan. In 1997 April, the Government of Japan increased the consumption tax from 3% to 5% and aggregate consumption spikes up before the tax hike and fall afterwards. Similarly, the Government of Japan increased the consumption tax from 5% to 8% in 2014 April and 8% to 10% in 2019 October. This aggregate level data shows the large impacts of the consumption tax changes on the aggregate consumptions.

Next, Figure 18 shows the non-durables and durable spendings since 2013 October until 2014 December. The data source of this figure 18 is Family Income and Expenditure Survey (FIES) which is collected in a monthly basis by the government to construct the price indices.<sup>35</sup>

Looking at the Figure 18, there are three noticeable features. First, the non-durable spendings are relatively stable over the sample periods, which is consistent to the theory. Second, the durable spendings spikes up right before the consumption tax hike. Third, the durable spendings after the tax hike is persistently lower than that of the initial level in the sample periods. These three findings are almost in line with the more cleanly identified pattern in Baker et al. (forthcoming) as discussed in the introduction with the Figure 1. Also, these findings are the facts which I

<sup>35</sup>In short, FIES is comparable to CEX in the U.S.

Figure 18: Non-Durable and Durable Spendings around 2014 April



Note: Taxable non-durable and durable spendings around 2014 April when the consumption tax is raised from 5% to 8%. Data source: FIES.

replicated using the baseline model.

## 12. Appendix C: Other Counter-Factuals

Since this paper develops the novel model for the policy analysis, it must be of certain interest to conduct a thought-experiments. This section discusses some experiments.

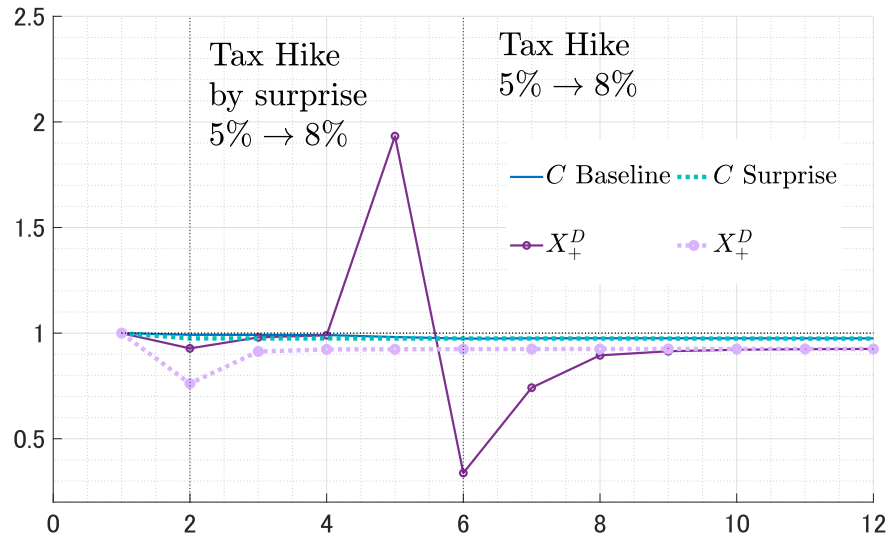
### 12.1. Unanticipated Tax Changes

This paper focused on the anticipated tax changes because it is the anticipation of the future tax hike that causes the stockpiling behavior of the durable goods. That is, if the consumption tax is suddenly implemented without a preannouncement, the households simply decrease the consumptions due to negative income effects. This subsection confirms this is the case.

Figure 19 shows the results of the experiment when the unanticipated tax hike is suddenly implemented in period 2 without the preannouncement. The unanticipated tax hike simply decreases the consumption levels ( $c, d$ ) in the period of the tax hike, and the expenditures stay constant after the tax hike  $t = 3, 4, \dots$

A clear difference from the anticipated tax change is that the unanticipated tax hike generate the larger negative income effects on the durable expenditure in periods 2. There are two reasons for this strong negative income effects. First, everyone face the reduction in the income and the

Figure 19: Anticipated Tax Hike vs Unanticipated Tax Hike



*Note:* When the tax hike is anticipated, the tax hike is preannounced in period 2 and implemented in period 6. When the tax hike is surprise (i.e., unanticipated), the tax hike is suddenly implemented in period 2 without the preannouncement. The unanticipated case exhibits the larger negative income effects.

asset when the tax hike is surprise. By contrast, when the tax hike is announced and implemented in the future, there is some old generations who pass away by the tax hike. These generations do not incur the negative income effects. Second, when the tax hike is announced, the stockpiling of the durable right before the tax hike is a way to mitigate the negative income effects. But, this measure is not available when the tax hike comes as the surprise.

## 12.2. Temporary Tax Cut: Germany in 2020

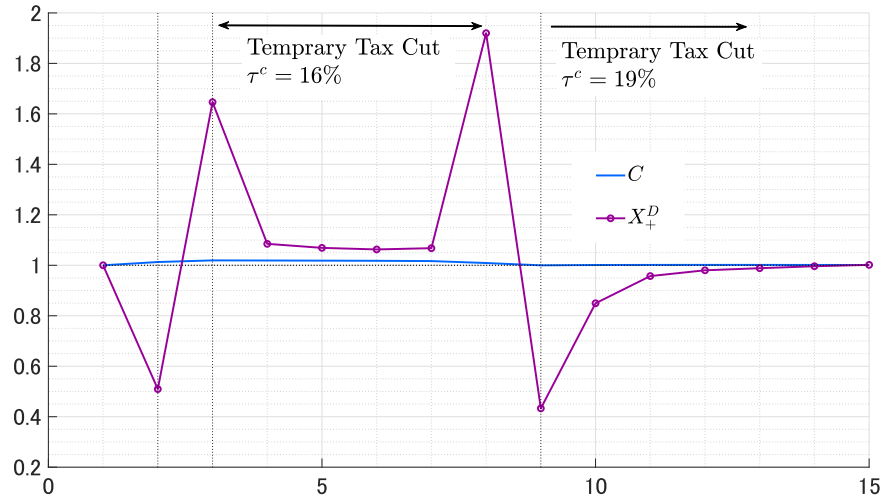
Next, I run an experiment designed to mimic the Germany's VAT cut implemented during the ongoing pandemic crisis. The German government announced in June 3rd that it would decrease the VAT rate by from 19% to 16% for six months between July and December.<sup>36</sup>

Figure 20 shows the results of the temporary tax cut experiment in a partial equilibrium in which tax cut from 19% to 16% is announced in period 2 and implemented between period 3 and period 9.

The result resembles the combination of the tax cut and hike. In the period 2 when tax cut is announced, the durable expenditure plummet since many households postpone the durable

<sup>36</sup>The reduced tax rate applicable to the grocery is also reduced by 2% in Germany. Also, the U.K. cut hospitality and tourism VAT by 15% for six months.

Figure 20: Temporary Tax Cut: Germany in 2020



*Note:* VAT is set 16% initially and economy is assumed in the stationary equilibrium in the first period. The government announces in period 2 that it would cut the tax between period 3 and period 9. Prices are fixed at stationary equilibrium levels.

purchase. Then, they refill the durable stock in the period 3. The aggregate durable expenditure remains higher than the stationary equilibrium level because of the temporary VAT cut. In the period 8, the households understand the temporary VAT cut ends in the next period and thus stock up the durables. The aggregate durable expenditure falls in the period 9 and gradually converges to the original stationary equilibrium afterwards.

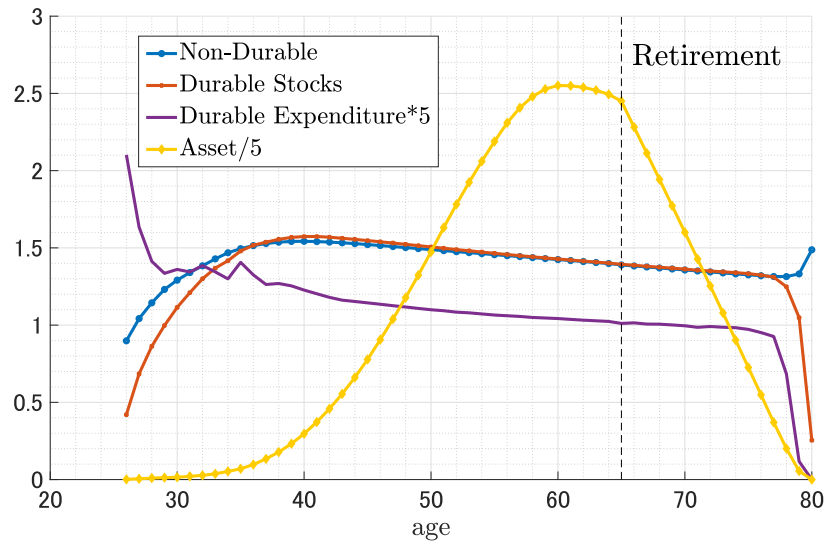
## 13. Appendix D: Sensitivity Check

### 13.1. Cobb-Douglas Utility Function

As discussed in the section 4, the Cobb-Douglas utility function is the most commonly used. Thus, it might be useful to show what if the Cobb-Douglas function is adopted.

Figure 21 shows the life-cycle profiles with the Cobb-Douglas utility function. There are three noteworthy differences from the Stone-Geary case. First, the durable expenditure profile with the Cobb-Douglas utility function is almost decreasing in age. This is because the new born households do not hold the durable stocks, have infinitely high marginal utility of durables, and thus purchase large amount of durables. Second, the consumption share between non-durables and durable stocks is almost stable after around  $j \geq 40$ . The constant share is directly coming

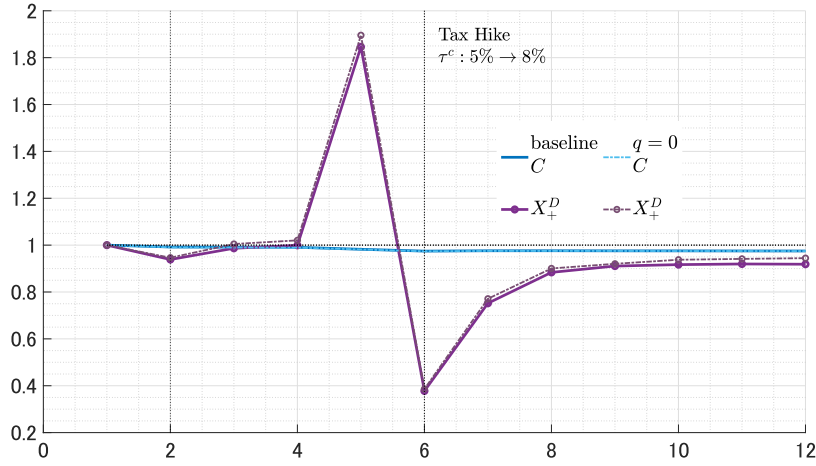
Figure 21: Life-cycle Profiles with Cobb-Douglas Utility



*Note:* There are three noticeable differences from the Stone-Geary utility. First, the durable expenditure profile with the Cobb-Douglas utility becomes almost decreasing in age. Second, the share between non-durable  $c$  and durable stock  $d$  becomes constant once the borrowing constraint becomes slack for most of households (above  $j \geq 40$ ). Third, the dying households  $j = 80$  hold plenty of durable stock on average.



Figure 22: Dynamics when  $q = 0$



Note: The dashed line shows the results of the baseline experiment with a parameter value  $q = 0$ . The baseline is when  $q = 1$ .

from the Cobb-Douglas nature  $u(c, d) = \frac{(c^\theta d^{1-\theta})^{1-\sigma}}{1-\sigma}$  where  $\theta$  governs the constant share as long as the borrowing constraint is not binding. Third, the very old households at age of 80 also own the positive amount of durable stocks with the Cobb-Douglas utility since Cobb-Douglas utility function satisfies Inada condition ( $\lim_{d \rightarrow 0} u_d(c, d) = \infty$ ).

Other change due to the Cobb-Douglas can be found over the dynamics. The Cobb-Douglas utility function, in general, generates less elastic dynamic responses in the aggregate durable expenditure since the Stone-Geary assumes the durable goods are luxury while the Cobb-Douglas does not. For example, the interest rate elasticity of the aggregate durable expenditure with the Stone-Geary is 4.1 while that of Cobb-Douglas is 0.6.

### 13.2. Choice of $q$

I set  $q = 1$  in the benchmark so as to maintain the model as much simple as possible. But this parameter choice of  $q$  may look ad-hoc. This section confirms that the choice of  $q$  have little influence on the results.

Figure 22 compares the baseline case  $q = 1$  and the other extreme calibration  $q = 0$ . This comparison confirms that the choice of  $q$  has little influence on the dynamics. Thus, I conclude that the choice of  $q$  is almost irrelevant for dynamic analysis.<sup>37</sup>

<sup>37</sup>I find the choice of  $q$  affects the share between the durables  $d$  and the assets  $a$ . The higher the  $q$  is, the durable play a role as a saving measure. Hence, the higher  $q$  leads to higher  $d$  and lower  $a$ .